

# Scattering of TE Wave on Screened Pre-Cantor Grating Based on Hypersingular Integral Equations

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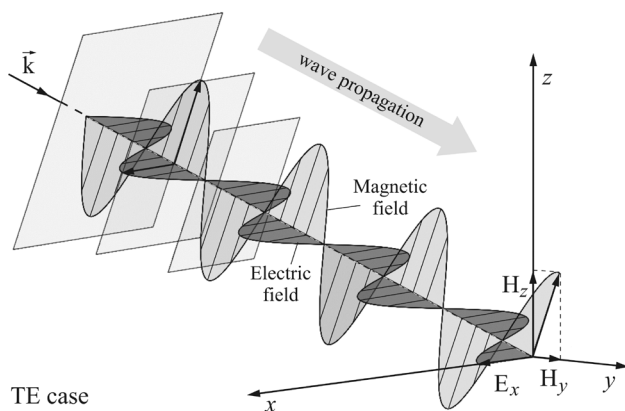
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**Abstract** In this paper a diffraction problem of a plane E-polarized wave on screened pre-Cantor grating is considered. A discrete mathematical model of this diffraction problem has been developed with the help of the specific quadrature formulas with the nodes in the nulls of Chebyshev polynomials. Using the parametric representations of hypersingular integral operator and integral operator with logarithmic singularity in the kernel, a hypersingular integral equation has been proposed. Besides, an effective discrete singularities method which has been applied to the numerical experiment of developed mathematical model has been proved to be one of the most efficient ways to solve such type of diffraction problems.

**Keywords** Hypersingular integral equation, Diffraction problem, Electromagnetic fields

## 1. Introduction

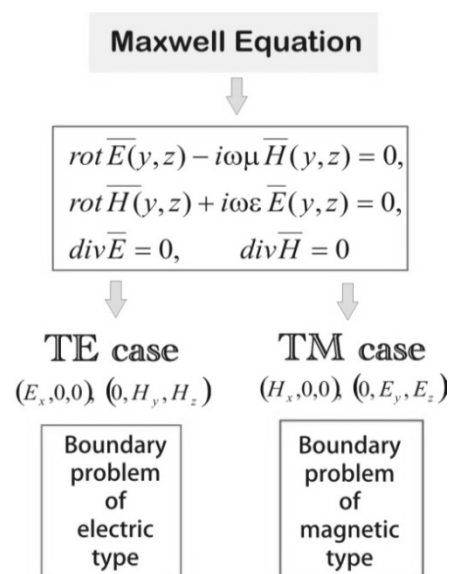
A transverse mode of a beam of electromagnetic radiation is perpendicular to the propagation direction of the beam. Transverse modes can be classified into two types: the transverse electric and transverse magnetic modes. If there is no magnetic field in the propagation direction then this is considered to be the TM case, while when there is no electric field in the direction of propagation then this is the TE case. In this paper the TE wave case is considered (see Fig. 1).



**Figure 1.** Propagation of the plane electromagnetic wave in TE case

Mathematical modelling of the interaction of electromagnetic radiation with plane-parallel electrodynamic structures is considered in many papers related to the

solution of applied problems. These structures include pre-Cantor perfectly conducting gratings above flat screen reflector. The key literature in this particular scientific field is formed by a monograph[1] and articles[2],[3], where boundary-value problems of diffraction mathematics theory are solved by considering boundary singular and hypersingular integral equations. In article[4] a mathematical and a discrete mathematical models of the diffraction problem on screened pre-Cantor grating for the case of H-polarization have been presented. Furthermore, a numerical experiment has been performance with the help of efficient discrete singularities method (DSM). In[5] a mathematical model of this diffraction problem was carried out.



**Figure 2.** Schematic structure to solve of 2D diffraction problem

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Figure 3. Constructing a pre-Cantor grating of the five order

The overall aim of this article is to develop a discrete mathematical model of hypersingular integral equations (HSIEs) using of the specific quadrature formulas with the nodes in the nulls of Chebyshev polynomials.

Novelty of this work is to choose the special strips in the form of pre-Cantor strips[11] for considering TE wave diffraction problem.

## 2. TE Diffraction Problem

To solve a two-dimensional (2-D) scattering problem, the total field which satisfies Maxwell's equations (see Fig. 2), boundary conditions on the strips and flat screen reflector, the Sommerfeld radiation conditions and the Meixner edge condition has been calculated.

Let us define the sets of pre-Cantor[11] intervals which are obtained by constructing a Cantor set on the  $N$ -th step - Fig. 3.

We consider the TE wave diffraction problem on multiple slits with finite numbers of non-equidistant strips where beginning and end of the strips are given by formula (1). Considering that the pre-Cantor strips are perfect electrically conducting (PEC). Propagation direction of a plane wave is given by direction of the wave vector  $\vec{k}$  (see Fig. 4).

Cartesian coordinate system is chosen so that the set of the strips is located in XY plane, and the X axis is parallel to the strips' edges (see Fig. 4).

$$Slits^{(N)} = \{(x, y, z) \in \mathbb{R} : x \in \mathbb{R}, y \in Sl_l^{(N)}, z = 0\},$$

$$Sl_l^{(N)} = \bigcup_{q=1}^{2^N-1} (a_q^N, b_q^N), \quad a_q = \left( \frac{P_q - 1}{2 \cdot 3^N} + \frac{1}{3^N} \right) 2l - 1, \quad (1)$$

$$b_q = \left( \frac{P_{q+1} - 1}{2 \cdot 3^N} \right) 2l - 1, \quad P_1 = 1, \quad q = 1, 2^N - 1,$$

$$P_{s+2^{k-1}} = 2 \cdot 3^k - P_{2^{k-1}-s+1}.$$

The time factor is assumed as  $e^{-i\omega t}$  and omitted.

The unique non-zero independent component of the electric field satisfies the two-dimensional Helmholtz equation above the flat screen reflector and without the strips:

$$\frac{\partial^2 E_x}{\partial y^2}(y, z) + \frac{\partial^2 E_x}{\partial z^2}(y, z) + \varepsilon \mu \omega^2 E_x(y, z) = 0, \quad (2)$$

$$\varepsilon \mu \omega^2 = k.$$

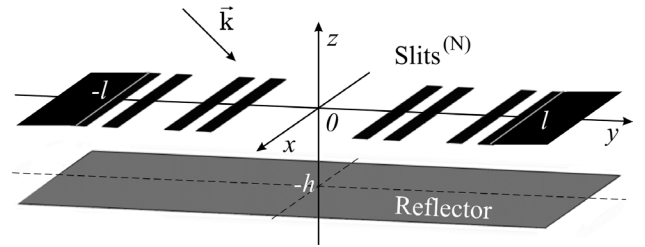


Figure 4. Schematic of the considered diffraction structure

Besides, this component satisfies all the aforementioned conditions and the unique non-zero independent components of the magnetic field are represented by Maxwell's equations:

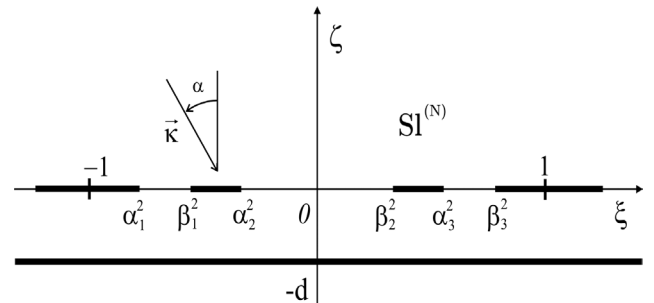
$$H_y(y, z) = \frac{1}{i\omega\mu} \frac{\partial E_x}{\partial z}(y, z), \quad (3)$$

$$H_z(y, z) = -\frac{1}{i\omega\mu} \frac{\partial E_x}{\partial y}(y, z).$$

For further convenience, the dimensionless coordinates are considered.

$$\varsigma = \frac{x}{l}, \xi = \frac{y}{l}, \zeta = \frac{z}{l}, \alpha_q^N = \frac{a_q^N}{l}, \beta_q^N = \frac{b_q^N}{l}, \quad (4)$$

$$d = \frac{h}{l}, \kappa = kl, Sl^{(N)} = \bigcup_{q=1}^{2^N-1} (\alpha_q^N, \beta_q^N).$$

Figure 5. Cross-section of the diffraction structure in the  $\xi\zeta$  plane

TE plane wave of unitary amplitude is incident from infinity onto the top of the diffraction structure (Fig. 5) at an angle  $\alpha$  :

$$u_{inc}^N(\xi, \zeta) = E_\zeta(\xi, \zeta) = e^{i\kappa(\xi \sin \alpha - \zeta \cos \alpha)}. \quad (5)$$

Here  $\alpha$  is an angle between positive direction of the  $\zeta$  axis and the propagation direction of a plane monochromatic wave, and  $\kappa$  is the absolute value of the wave vector.

The total electric field has only  $\zeta$  component  $u^{(N)}(\xi, \zeta) = E_\zeta(\xi, \zeta)$  which can be represented as:

$$u^{(N)}(\xi, \zeta) = \begin{cases} u_0^N(\xi, \zeta) + u_+^N(\xi, \zeta), & \zeta > 0, \\ u_-^N(\xi, \zeta), & -d < \zeta < 0, \end{cases} \quad (6)$$

where  $u_0^N(\xi, \zeta)$  is a known solution of the Helmholtz equation when the slits are closed. The functions  $u_+^N(\xi, \zeta)$  and  $u_-^N(\xi, \zeta)$  are to be determined.

The total field should satisfy the following conditions:

1. the Helmholtz equation above the flat screen reflector outside of the strips

$$\Delta u^{(N)}(\xi, \zeta) + \kappa u^{(N)}(\xi, \zeta) = 0; \quad (7)$$

2. the boundary conditions on the strips:

$$u^{(N)}(\xi, \zeta) \Big|_{\zeta=0} = 0, \quad \xi \in CSI^{(N)} = \Re \setminus SI^{(N)}, \quad (8)$$

and on the flat screen reflector:

$$u^{(N)}(\xi, \zeta) \Big|_{\zeta=-d} = 0, \quad \xi \in CSI^{(N)}; \quad (9)$$

3. the conditions of conjugation in the slits:

$$u^{(N)}(\xi, +0) = u^{(N)}(\xi, -0), \quad \xi \in SI^{(N)}, \quad (10)$$

$$\frac{\partial u^{(N)}}{\partial \zeta}(\xi, +0) = \frac{\partial u^{(N)}}{\partial \zeta}(\xi, -0), \quad \xi \in SI^{(N)}; \quad (11)$$

4. the Sommerfeld radiation conditions:

$$u_{scat}^{(N)} = \mathcal{O}\left(\frac{1}{\sqrt{r}}\right), \quad \frac{\partial u_{scat}^{(N)}}{\partial r} - i\kappa u_{scat}^{(N)} = \mathcal{O}\left(\frac{1}{\sqrt{r}}\right), \quad r \rightarrow \infty, \quad (12)$$

where  $\vec{r}$  is the vector radius of the observation point;

5. the Meixner edge condition (the local energy finiteness in any finite domain enclosing an edge point):

$$\int_{\Omega} \left\{ \kappa^2 |u_{\pm}^N(x, y)|^2 + |\nabla u_{\pm}^N(x, y)|^2 \right\} d\sigma < \infty, \quad (13)$$

where  $\Omega$  is any boundary field in  $\Re^2$ .

The function  $u_0^N(\xi, \zeta)$  has the form:

$$u_0^N(\xi, \zeta) = e^{i\kappa(\xi \sin \alpha - \zeta \cos \alpha)} - e^{i\kappa(\xi \sin \alpha + \zeta \cos \alpha)}. \quad (14)$$

And functions  $u_+^N(\xi, \zeta)$ ,  $u_-^N(\xi, \zeta)$  can be written as Fourier representation:

$$u_+^N(\xi, \zeta) = \int_{-\infty}^{+\infty} C_+^N(\lambda) e^{i\lambda \xi - \gamma(\lambda) \zeta} d\lambda, \quad (15)$$

$$u_-^N(\xi, \zeta) = \int_{-\infty}^{+\infty} C_-^N(\lambda) \frac{sh(\gamma(\lambda)(\zeta + d))}{sh(\gamma(\lambda)d)} e^{i\lambda \xi} d\lambda. \quad (16)$$

The radiation condition is satisfied if  $\gamma(\lambda) = \sqrt{\lambda^2 - \kappa^2}$ ,  $\lambda \in \Re$ , and is given by  $\text{Re}(\gamma(\lambda)) \geq 0$ ,  $\text{Im}(\gamma(\lambda)) \leq 0$ ,  $\lambda \in \Re$ .

Such a scattering problem is uniquely solvable for the function  $u^{(N)}(\xi, \zeta)$ .

We need to find the total field (6) which results from the scattering of E polarized monochromatic plane wave on considered diffraction structure.

### 3. Hypersingular Integral Equation

Hypersingular integral equations[9], i.e., integral equations whose kernel has a singularity of an order greater than one (in our case the order singularity is two). HSIE is understood in the sense of finite part by Hadamard[8],[9].

Substituting the expressions of functions (15) and (16) into the boundary conditions (8), (9), and into the conditions of conjugation in the slits (10), (11). Thus, obtain four equations: two into strips and two into slits. Using these equations and formula (14) the equality for coefficients  $C_+^N(\lambda) = C_-^N(\lambda)$  has been received for every lambda and a coupled integral equation should be written as:

$$\begin{cases} \int_{-\infty}^{+\infty} C^N(\lambda) e^{i\lambda \xi} d\lambda = 0, & \xi \in CSI^{(N)}, \\ \int_{-\infty}^{+\infty} \gamma(\lambda) C^N(\lambda) (1 + \text{cth}(\gamma(\lambda))) e^{i\lambda \xi} d\lambda = f(\xi), & \xi \in SI^{(N)}, \end{cases} \quad (17)$$

where

$$f(\xi) = \frac{1}{2} \frac{\partial u_0^N}{\partial \xi}(\xi, +0), \quad C^N(\lambda) = C_+^N(\lambda) = C_-^N(\lambda).$$

Following expression is introducing a new unknown function:

$$F^N(\eta) = \int_{-\infty}^{+\infty} C^N(\lambda) e^{i\lambda \eta} d\lambda, \quad \eta \in \Re. \quad (18)$$

From the formula (17) following that,

$$F^N(\xi) = 0, \quad \xi \in CSI^{(N)}. \quad (19)$$

Using the inverse Fourier transform, a relation to the unknown coefficient can be defined:

$$C^N(\lambda) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F^N(\xi) e^{-i\lambda\xi} d\xi. \quad (20)$$

Denote,

$$\begin{aligned} \tilde{W}(\lambda) &= \gamma(\lambda)(1 + cth(\gamma(\lambda)d)), \\ W(\lambda) &= 2 \left( |\lambda| - \frac{\kappa^2}{2\gamma(\lambda)} + \frac{\kappa^4}{2\gamma(\lambda)(|\lambda| + \gamma(\lambda))^2} \right), \quad (21) \\ W(\lambda) &= \tilde{W}(\lambda) + \underline{Q} \left( \frac{1}{\lambda} \right), \quad \Delta(\lambda) = \tilde{W}(\lambda) - W(\lambda). \end{aligned}$$

Using following relation

$$\lim_{\lambda \rightarrow \infty} (1 + cth(\gamma(\lambda)d)) = 2$$

and formula (21), a second equation from (17) has the form:

$$\begin{aligned} \int_{-\infty}^{+\infty} W(\lambda) C^N(\lambda) e^{i\lambda\xi} d\lambda + \int_{-\infty}^{+\infty} \Delta(\lambda) C^N(\lambda) e^{i\lambda\xi} d\lambda &= \\ &= f(\xi), \quad \xi \in SI^{(N)}. \end{aligned} \quad (22)$$

Use the expansion

$$\begin{aligned} \frac{\kappa^2 i}{2} \int_{-\infty}^{+\infty} H_0^{(1)}(\kappa|\eta - \xi|) F^N(\eta) d\eta &= \\ &= \frac{\kappa^2}{\pi} \int_{SI^{(N)}} \ln|\eta - \xi| F^N(\eta) d\eta + \frac{\kappa^2}{\pi} \int_{SI^{(N)}} K(\eta, \xi) F^N(\eta) d\eta, \end{aligned} \quad (23)$$

where

$$K(\eta, \xi) = \frac{\pi i}{2} H_0^{(1)}(\kappa|\eta - \xi|) - \ln|\eta - \xi|. \quad (24)$$

To lead the coupled integral equation (17) into hypersingular integral equations it is necessary to use specific parametric representations of the following form:

• parametric representations[3] of hypersingular integral operator:

$$\begin{cases} F^N(\eta) = \int_{-\infty}^{+\infty} C^N(\lambda) e^{i\lambda\eta} d\lambda, \\ \frac{1}{\pi} \int_{SI^{(N)}} \frac{F^N(\eta)}{(\eta - \xi)^2} d\eta = - \int_{-\infty}^{+\infty} |\lambda| C^N(\lambda) e^{i\lambda\xi} d\lambda, \end{cases} \quad (25)$$

• integral operator with logarithmic singularity in the kernel:

$$\begin{cases} F^N(\eta) = \int_{-\infty}^{+\infty} C^N(\lambda) e^{i\lambda\eta} d\lambda, \\ \frac{i}{2} \int_{SI^{(N)}} H_0^{(1)}(\kappa|\eta - \xi|) F^N(\eta) d\eta = - \int_{-\infty}^{+\infty} \frac{C^N(\lambda)}{\gamma(\lambda)} e^{i\lambda\xi} d\lambda. \end{cases} \quad (26)$$

From the Equation (22) using formulas (23) - (26), the hypersingular integral equation can be obtained:

$$\begin{aligned} \frac{1}{\pi} \int_{SI^{(N)}} \frac{F^N(\eta)}{(\eta - \xi)^2} d\eta - \frac{\kappa^2}{\pi} \int_{SI^{(N)}} \ln|\eta - \xi| F^N(\eta) d\eta + \\ + \frac{1}{\pi} \int_{SI^{(N)}} Q(\eta, \xi) F^N(\eta) d\eta = g(\xi), \quad \xi \in SI^{(N)}, \end{aligned} \quad (27)$$

where

$$\begin{aligned} Q(\eta, \xi) &= -\frac{1}{2} \left( \kappa^2 \frac{\pi i}{2} H_0^{(1)}(\kappa|\eta - \xi|) - \kappa^2 \ln|\eta - \xi| + \right. \\ &\left. + \frac{1}{2} \int_{-\infty}^{+\infty} \left( \frac{\kappa^4}{\gamma(\lambda)(|\lambda| + \gamma(\lambda))^2} + \Delta(\lambda) \right) e^{i\lambda(\xi - \eta)} d\lambda \right), \quad g(\xi) = -\frac{f(\xi)}{2}. \end{aligned} \quad (28)$$

Mathematical model in equation (27) describes the solution for diffraction problem on screened pre-Cantor grating and is based on hypersingular integral equations.

Introduce the restrictions of functions  $g_p^N(\xi)$ ,  $F_q^N(\eta)$ , on intervals

$$SI_q^{(N)} = (\alpha_q^N, \beta_q^N), \quad q = 1, 2^N - 1:$$

$$g_p^N(\xi) = g^N(\xi) \Big|_{\xi \in SI_p^{(N)}}, \quad F^N(\eta) = F_q^N(\eta) \Big|_{\eta \in SI_q^{(N)}}. \quad (29)$$

Thus, equation (27) can be written in the form for  $p = 1, 2^N - 1$ :

$$\begin{aligned} \frac{1}{\pi} \sum_{q=1}^{2^N-1} \int_{\alpha_q^N}^{\beta_q^N} \frac{F_q^N(\eta) d\eta}{(\eta - \xi)^2} - \\ - \frac{\kappa^2}{\pi} \sum_{q=1}^{2^N-1} \int_{\alpha_q^N}^{\beta_q^N} \ln|\eta - \xi| F_q^N(\eta) d\eta + \\ + \frac{1}{\pi} \sum_{q=1}^{2^N-1} \int_{\alpha_q^N}^{\beta_q^N} Q(\eta, \xi) F_q^N(\eta) d\eta = g_p^N(\xi), \quad \xi \in SI_p^{(N)}. \end{aligned} \quad (30)$$

For the case if  $p=q$ :

$$P_{q,p}^N(\eta, \xi) = \begin{cases} Q(\eta, \xi), & p = q, \\ \frac{1}{(\eta - \xi)^2} - \kappa^2 \ln |\eta - \xi| + Q(\eta, \xi), & p \neq q, \end{cases}$$

therefore, equation (30) can be transformed to:

$$\begin{aligned} & \frac{1}{\pi} \int_{\alpha_p^N}^{\beta_p^N} \frac{F_p^N(\eta) d\eta}{(\eta - \xi)^2} - \frac{\kappa^2}{\pi} \int_{\alpha_p^N}^{\beta_p^N} \ln |\eta - \xi| F_p^N(\eta) d\eta + \\ & \frac{1}{\pi} \sum_{q=1}^{2^N-1} \int_{\alpha_q^N}^{\beta_q^N} P_{q,p}^N(\eta, \xi) F_q^N(\eta) d\eta = g_p^N(\xi), \quad \xi \in SI_p^{(N)}. \end{aligned} \quad (31)$$

Standard interval  $(-1, 1)$  has been chosen and is displayed to the intervals:

$$\begin{aligned} SI_q^{(N)} &= (\alpha_q^N, \beta_q^N), \quad q = 1, \dots, 2^N - 1, \\ g^{(N)} : (-1, 1) &\rightarrow (\alpha_q^N, \beta_q^N) : \\ t &\rightarrow g^{(N)}(t) = \frac{\beta_q^N - \alpha_q^N}{2} t + \frac{\beta_q^N + \alpha_q^N}{2}, \end{aligned} \quad (32)$$

make the substitution for  $q = 1, \dots, 2^N - 1$ ,

$$\begin{aligned} \eta &= g^{(N)}(t), \quad \xi = g^{(N)}(t_0), \quad t, t_0 \in [-1, 1], \\ \eta &\in (\alpha_q^N, \beta_q^N), \quad \xi \in (\alpha_p^N, \beta_p^N). \end{aligned} \quad (33)$$

After simple transformations from (31) using (32), (33), HSIEs on standard intervals can be obtained:

$$\begin{aligned} & \frac{2}{\beta_p^N - \alpha_p^N} \frac{1}{\pi} \int_{-1}^1 \frac{F_p^N(g^{(N)}(t))}{(t - t_0)^2} dt - \\ & - \frac{\kappa^2}{\pi} \frac{\beta_p^N - \alpha_p^N}{2} \int_{-1}^1 \ln |t - t_0| F_p^N(g^{(N)}(t)) dt + \\ & + \frac{1}{\pi} \frac{\beta_p^N - \alpha_p^N}{2} \sum_{q=1}^{2^N-1} \int_{-1}^1 P_{q,p}^N(g^{(N)}(t), g^{(N)}(t_0)) \times \\ & \times F_q^N(g^{(N)}(t)) dt = g_p^N(g^{(N)}(t_0)), \end{aligned} \quad (34)$$

where  $P_{q,p}^N(g^{(N)}(t), g^{(N)}(t_0))$ ,  $g_p^N(g^{(N)}(t_0))$  are smooth known functions:

$$\begin{aligned} P_{q,p}^N(g^{(N)}(t), g^{(N)}(t_0)) &= \\ &= \begin{cases} Q(g^{(N)}(t), g^{(N)}(t_0)) - \ln \left| \frac{\beta_p^N - \alpha_p^N}{2} \right|, & p = q, \\ \frac{1}{(g^{(N)}(t) - g^{(N)}(t_0))^2} - \\ - \kappa^2 \ln |g^{(N)}(t) - g^{(N)}(t_0)| + \\ + Q(g^{(N)}(t), g^{(N)}(t_0)), & p \neq q. \end{cases} \end{aligned}$$

The Meixner edge condition will be satisfied if the unknown function (18) is represented in the form:

$$F_q^N(g^{(N)}(t)) = v_q^N(g^{(N)}(t)) \sqrt{1 - t^2}, \quad |t| < 1, \quad (35)$$

then equation (34) have a subsequent representation:

$$\begin{aligned} & \frac{1}{\pi} \frac{2}{\beta_p^N - \alpha_p^N} \int_{-1}^1 \frac{v_p^N(g^{(N)}(t))}{(t - t_0)^2} \sqrt{1 - t^2} dt - \\ & - \frac{\kappa^2}{2} (\beta_p^N - \alpha_p^N) \frac{1}{\pi} \int_{-1}^1 \ln |t - t_0| v_p^N(g^{(N)}(t)) \sqrt{1 - t^2} dt + \\ & + \frac{1}{\pi} \sum_{q=1}^{2^N-1} \frac{(\beta_p^N - \alpha_p^N)}{2} \int_{-1}^1 P_{q,p}^N(g^{(N)}(t), g^{(N)}(t_0)) \times \\ & \times v_q^N(g^{(N)}(t)) \sqrt{1 - t^2} dt = g_p^N(g^{(N)}(t_0)), \quad |t_0| < 1, \end{aligned} \quad (36)$$

where  $P_{q,p}^N(g^{(N)}(t), g^{(N)}(t_0))$ ,  $g_p^N(g^{(N)}(t_0))$  are smooth known functions.

## 4. Discrete Mathematical Model

We can continue to improve our model when new information comes in. The most important step is numerical experiments which depicted some interesting phenomena. For example, the near-field has been of increasing interest, particularly in the development of capacitive sensing technologies such as those used in smart phones and tablet computers. And many other characteristics and parameters are important for future investigation of this particular task. Furthermore, discrete mathematics is the mathematical language of computer science, and as such, its importance has increased dramatically in recent decades. This is due to the fact that there are an increasing number of new software solutions to specific problems. For this reason, it is necessary to lead mathematical model (36) using discretization into discrete mathematical model for perform a numerical analysis.

We build a discrete mathematical model based on equation (36) as follows. The first step is interpolated by a Lagrange polynomial of order  $(n-2)$  in the nodes which are the nulls of Chebyshev polynomials of the second kind[8]:

$$t_{0j}^n = \cos\left(\frac{j}{n}\pi\right), \quad j = \overline{1, n-1},$$

of the unknown function

$$v_q^N(g_q^{(N)}(t))$$

and smooth function

$$P_{q,p}^N(g_q^{(N)}(t), g_p^{(N)}(t_0)), \quad g^N(g_p^{(N)}(t_0)).$$

On the next step, the specific quadrature formulas[8] of interpolation type with the equidistant grid of the polynomial of order  $(n-2)$  for integrals with hypersingular and logarithmic singularities and integrals of smooth functions are considered.

$$\begin{aligned} & \frac{1}{\pi} \int_{-1}^1 \ln |t - t_0| p_{n-2}(t) \sqrt{1-t^2} dt = \\ & = -\frac{1}{n} \sum_{j=1}^{n-1} p_{n-2}(t_{0j}^n) \left(1 - (t_{0j}^n)^2\right) \times \\ & \times \left[ \ln 2 + 2 \sum_{k=1}^{n-1} \frac{T_k(t_{0j}^n)}{k} T_k(t_0) + \frac{(-1)^j}{n} T_k(t_0) \right], \\ & \frac{1}{\pi} \int_{-1}^1 \frac{p_{n-2}(t)}{(t - t_{0k}^n)^2} \sqrt{1-t^2} dt = \\ & \sum_{\substack{j=1, \\ j \neq k}}^{n-1} p_{n-2}(t_{0j}^n) \frac{(1 - (-1)^{j+k})(1 - (t_{0j}^n)^2)}{(t_{0k}^n - t_{0j}^n)^2} \frac{1}{n} - p_{n-2}(t_{0k}^n) \frac{n}{2}. \end{aligned} \quad (37)$$

Interpolated polynomials for all functions in this problem coincide with initial functions in the node points:

$$\begin{aligned} & \left(P_{n-2}^{II} v_q^{N, (n-2)}\right)(t_{0j}^n) = v_q^{N, (n-2)}(t_{0j}^n), \quad j = \overline{1, n-1}, \\ & \left(P_{n-2}^{II} P_{q,p}^{N, (n-2), (n-2)}\right)(t_{0j}^n, t_{0k}^n) = P_{q,p}^{N, (n-2), (n-2)}(t_{0j}^n, t_{0k}^n), \\ & j = \overline{1, n-1}, \\ & \left(P_{n-2}^{II} g_p^{N, (n-2)}\right)(t_{0k}^n) = g_p^{N, (n-2)}(t_{0k}^n), \quad k = \overline{1, n-1}. \end{aligned} \quad (38)$$

In such a way a system for approximate solutions from the mathematical model (36) is obtained. With the help of an efficient numerical discrete singularities method we got the system of linear algebraic equations for

$$p = \overline{1, 2^N - 1}, \quad k = \overline{1, n-1} :$$

$$\left\{ \begin{aligned} & \frac{2}{\beta_p^N - \alpha_p^N} \sum_{\substack{j=1, \\ j \neq k}}^{n-1} v_p^{N,(n-2)}(g^N(t_{0j}^n)) \times \\ & \times \frac{(1 - (-1)^{j+k})(1 - (t_{0j}^n)^2)}{(t_{0k}^n - t_{0j}^n)^2} \frac{1}{n} - \\ & - \frac{n \cdot v_p^{N,(n-2)}(g^N(t_{0k}^n))}{\beta_p^N - \alpha_p^N} + \\ & + \frac{\kappa^2}{2} (\beta_p^N - \alpha_p^N) \frac{1}{n} \sum_{j=1}^{n-1} v_p^{N,(n-2)}(g^N(t_{0j}^n)) \times \\ & \times (1 - (t_{0j}^n)^2) \left[ \ln 2 + 2 \sum_{k=1}^{n-1} \frac{T_k(t_{0j}^n)}{k} T_k(t_0) + \frac{(-1)^j}{n} T_k(t_0) \right] + \\ & + \sum_{q=1}^{2^N-1} \frac{\beta_q^N - \alpha_q^N}{2} \sum_{k=1}^{n-1} P_{q,p}^{N,(n-2),(n-2)}(g^N(t_{0j}^n), g^N(t_{0k}^n)) \times \\ & \times v_q^{N,(n-2)}(g^N(t_{0j}^n)) = g_p^{N,(n-2)}(g^N(t_{0k}^n)), \end{aligned} \right. \quad (39)$$

where

$$P_{q,p}^{N,(n-2),(n-2)}(g^N(t_{0j}^n), g^N(t_{0k}^n)), \quad g_p^{N,(n-2)}(g^N(t_{0k}^n))$$

are smooth known functions.

By solving the linear system (39) we find the values of unknown functions in the node points and calculate the unknown coefficient (20) to obtain the scattered and diffracted fields (15), (16). Using the asymptotic representation of the Hankel functions [12], [13] and functions in (15), (16), we derive the expressions for far field radiation patterns of the scattered and diffracted fields.

The quality of the antenna is determined by its characteristics and parameters. Most antennas have directional properties. These properties are characterized by the radiation pattern. In consequence of this fact, necessity to calculate of the radiation pattern is major part of the numerical analysis.

The analysis of various electromagnetic phenomena can be a very complicated task from the experimental point of view. The use of mathematical modelling, however, allows simulation of complex cases and sometimes is the only tool at hand of the engineer.

In papers [6], [7], the numerical results for scattering problems on pre-Cantor impedance strips in cases of the TE and TM waves based on singular integral equations are

discussed.

## 5. Conclusions

In this paper, a discrete mathematical model of the TE wave diffraction problem on screened pre-Cantor grating has been proposed. Mathematical model of this task is based on hypersingular integral equations. In order to resolve the HSIEs it is essential to use specific quadrature formulas for integrals with singularities. Investigation result of this work showed that the all difficulties with singularities in integrals are solved. Thus, it can be concluded that the overall aim of this article has been achieved. Discrete mathematical model of the diffraction problem on screened grating has been carried out. As show in the findings, the present investigations are actual and in-demand. Based on the obtained discrete mathematical model, the numerical experiments will be calculated with the help of an efficient discrete singularities method based on created discrete mathematical model. This experiment will be illustrated a main properties of scattered and diffracted fields such as

diffraction and radiation patterns, field dependence on different characteristics and other. Future prospects in this field are investigation of 3-D diffraction problems on pre-fractal grating and carrying out numerical experiments.

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