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(1,2) - Domination in Line Graphs of C_n, P_n and $K_{1,n}$

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Abstract In this paper we discuss the (1,2) - domination in line graphs of C_n, P_n and $K_{1,n}$.(1,2)-domination number of paths, cycles, star graphs are found and compared it with the usual domination number. Also we find the domination number of line graphs of these graphs.

Keywords Dominating Set, Domination Number, (1,2) - dominating Set, (1,2) - domination Number

1. Introduction

Let G = (V,E) be a simple graph. A subset D of V is a *dominating set* of G if every vertex of V- D is adjacent to a vertex of D. The *domination number* of G, denoted by $\gamma(G)$, is the minimum cardinality of a dominating set of G.[1,2]

A (1,2) - dominating set in a graph G = (V,E) is a set S having the property that for every vertex v in V - S there is atleast one vertex in S at distance 1 from v and a second vertex in S at distance almost 2 from v. The order of the smallest (1,2) - dominating set of G is called the *(1,2) - domination number* of G and we denote it by $\gamma_{(1,2)}$.[7]

The line graph L(G) of a graph G = (V,E) is a graph with vertex set E(G) in which two vertices are adjacent if and only if the corresponding edges in G are adjacent.[8]

2. (1,2) - domination in Paths

Throughout this paper P_n denotes a path on n vertices, C_n denotes a cycle with n vertices and $K_{1,n}$ is a star graph.

Consider the following paths

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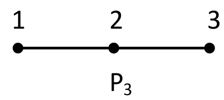
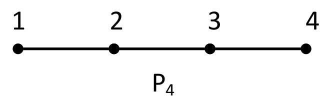


{2,3} is a (1,2) - dominating set.
$\gamma_{(1,2)} = 2$



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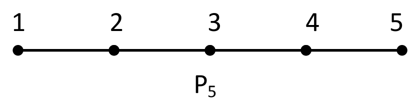
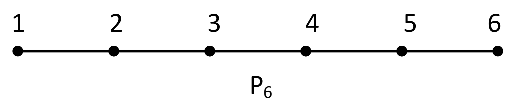
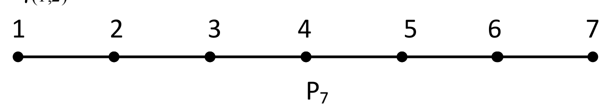


{2,3,4 } is a (1,2) - dominating set.
$\gamma_{(1,2)} = 3$



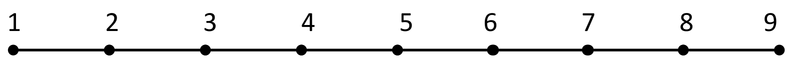
{2,3,4,5} is a (1,2) - dominating set.
$\gamma_{(1,2)} = 4$



{2,3,4,5,6} is a (1,2) - dominating set.
$\gamma_{(1,2)} = 5$

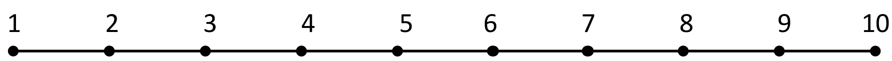
P_8

{2,3,4,5,6,7} is a (1,2) - dominating set.
$\gamma_{(1,2)} = 6$



P_9

{2,3,4,5,6,7,8} is a (1,2) - dominating set.
$\gamma_{(1,2)} = 7$



P_{10}

{2,3,4,5,6,7,8,9} is a (1,2) - dominating set.
$\gamma_{(1,2)} = 8$

From the above examples we have the following theorem.

Theorem 2.1

(1,2) - domination number of a path graph P_n, for $n \geq 4$ is n-2.

Proof:

Let P_n be a path with n vertices $v_1, v_2, ..., v_n$. Then $v_2, v_3, ..., v_{n-1}$ are of degree 2 ,v_1 and v_2 are of degree 1. That is n-2 vertices are of degree 2. Each vertex v_1 is adjacent to v_{i+1}. Therefore, v_i's are at distance one from v_{i+1}. Each vertex v_{i+2} is at distance 2 from v_i. So to form a (1,2) - dominating set we have to include all those vertices of degree 2. But there are n-2 vertices of degree 2.

Hence $\gamma_{(1,2)} = n-2$.

The following lemma is due to Gray Chartrand and Ping Zhang.[3]

Lemma 1. $\gamma(P_n) = \left\lceil \frac{n}{3} \right\rceil$

The following theorem gives the relationship between domination number and (1,2)-domination number.

Theorem 2.2

The domination number of the path P_n is less than (1,2) domination number.

Proof:

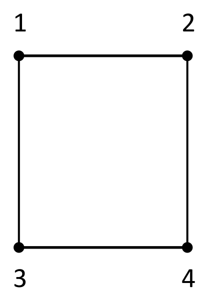
We have $\gamma(P_n) = \left\lceil \frac{n}{3} \right\rceil$

In a graph G, domination number is less than or equals (1,2) domination number[6]. Let G be a graph and D be its dominating set. Then every vertex in V-D is adjacent to a vertex in D. That is, in D, for every vertex u, there is a vertex which is at distance 1 from u. But it is not necessary that there is a second vertex at distance atmost 2 from u. So if we find a (1,2)- dominating set, it will contain more vertices or atleast equal number of vertices than the dominating set. So the domination number is less than or equal to (1,2)-domination number. In particular, for paths domination number is less than(1,2) domination number. Hence the theorem.

3. (1,2)-domination in Cycles

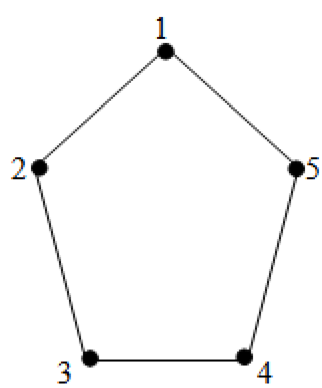
Consider the following cycles

C_4 :



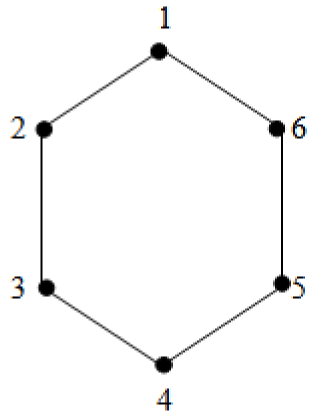
{2,3} is a (1,2) - dominating set.
$\gamma_{(1,2)} = 2$

C_5 :



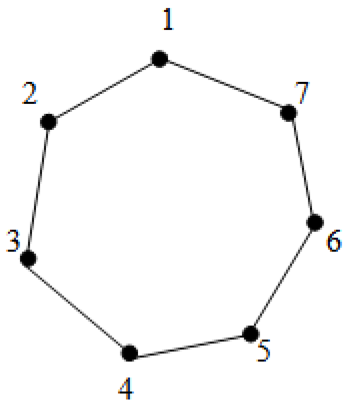
{1,3,4} is a (1,2) - dominating set.
$\gamma_{(1,2)} = 3$

C_6 :



{1,2,4} is a (1,2) - dominating set. $\gamma_{(1,2)} = 3$

C_7 :



{1,3,4,7} is a (1,2) - dominating set.
$\gamma_{(1,2)} = 4$

From the above examples we have the following theorem.

Theorem 3.1

For cycle C_n, $n \geq 4$ $\quad \gamma_{(1,2)} = \begin{cases} \frac{n}{2}, \text{if n is even} \\ n-2, \text{if n is odd} \end{cases}$

Proof:

Every cycle C_n have n vertices and n edges in which each vertex is of degree 2. That is each vertex dominates two vertices .

Case 1: n is even.

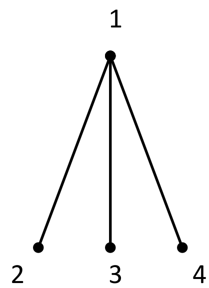
Let $v_1, v_2, ..., v_n$ be the vertices of C_n. v_1 and v_n are adjacent. Also v_1 is adjacent to v_2, v_2 is adjacent to v_3 and so on. Each v_i, $1 < i < n$ is not adjacent to $v_{i+2}, v_{i+3}, v_{i+4}, ..., v_{n-1}$. Let us construct a (1,2) - dominating set. If we take the vertex v_1, v_1 is adjacent to v_2 and v_n and non-adjacent to all other n-2 vertices. v_3 and v_{n-1} are at distance 2 from v_1. So we have to take v_2 and any one of v_3, v_{n-1} in the set. If we take v_2, v_2 is adjacent to v_1 and v_3 and non- adjacent to $v_4, v_5, ... v_n$. That is n-3 vertices and v_4 and v_{n-2} are at distance 2 from v_2. Similarly we can proceed upto all the n vertices. Finally we get a (1,2)- dominating set containing $v_1, v_2, v_4, v_6, .., v_{n-2}$. The cardinality of this set is $1+\frac{n}{2}-1$, that is $\frac{n}{2}$. Hence $\gamma_{(1,2)}$ $= \frac{n}{2}$, if n is even

Case 2: n is odd

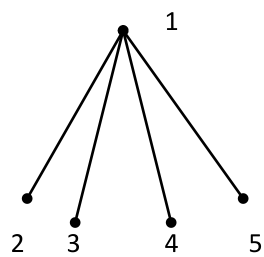
If n is odd, we remove one vertex v_1, then the other n-1 vertices form a path P_{n-1} and n-1 is even. But (1,2) - domination number of P_{n-1} is n- 3. These n-3 vertices and the vertex v_1 from a (1,2) dominating set. Hence the cardinality of the (1,2) dominating set is n- 3+1. that is n-2. Hence $\gamma_{(1,2)}$ = n-2, if n is odd.

4. (1,2)-domination in Star Graphs

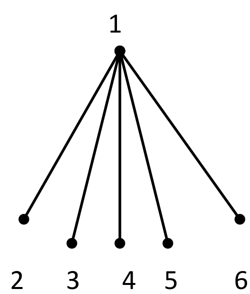
Consider the following star graphs.



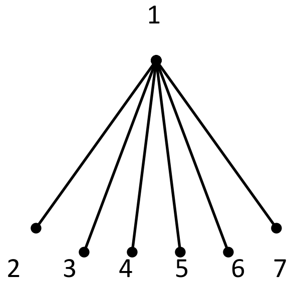
{1,2} form a (1,2) - dominating set.
$\gamma_{(1,2)} = 2$



{1,2} form a (1,2) - dominating set.
$\gamma_{(1,2)} = 2$



{1,2} form a (1,2) - dominating set. $\gamma_{(1,2)} = 2$



{1,2} form a (1,2) - dominating set.
$\gamma_{(1,2)} = 2$

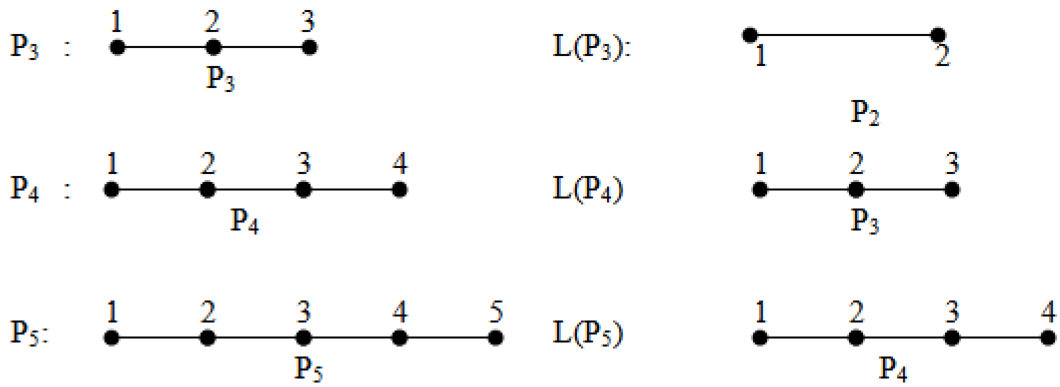
Theorem 4.1
For any star $K_{1,n}$, $\gamma_{(1,2)}(K_{1,n}) = 2$.

Proof:
In a star $K_{1,n}$, there are n+1 vertices $v, v_1, v_2, ..., v_n$. v is adjacent to all other vertices $v_1, v_2, ..., v_n$. $\{v_1, v_2, ..., v_n\}$ form an independent set. Each of $v_1, v_2, ..., v_n$ are at a distance 1 from v and each of $v_2, v_3, ..., v_n$ are at a distance 2 from v_1. So we can form a (1,2) dominating set as $\{v, v_1\}$. Clearly the cardinality is two. Hence $\gamma_{(1,2)}(K_{1,n}) = 2$.

5. (1,2) - domination in the Line Graph of P_n, C_n, $K_{1,n}$.

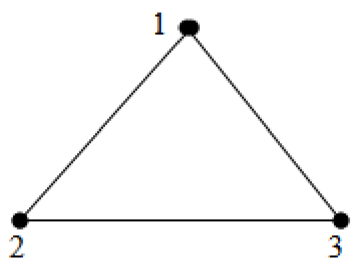
In this section first we discuss the line graphs of paths, cycles and star graphs.

Consider the paths and the corresponding line graphs

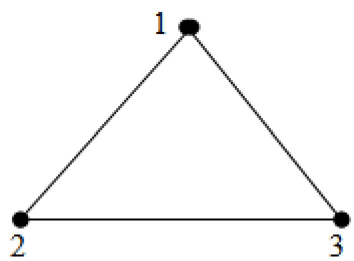


Next consider the cycles and the corresponding line graphs

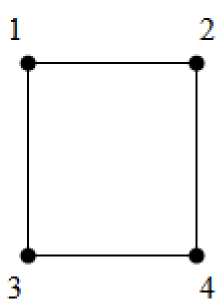
C_3:



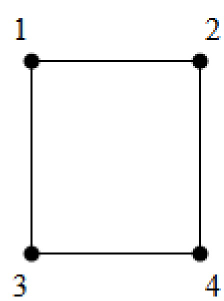
$L(C_3)$



C_4:

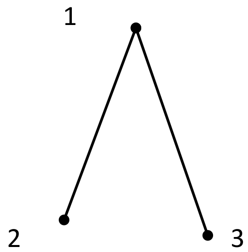


$L(C_4)$

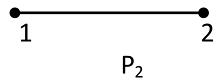


Consider the following star graphs and the corresponding line graphs

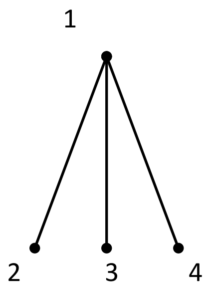
$K_{1,2}$:



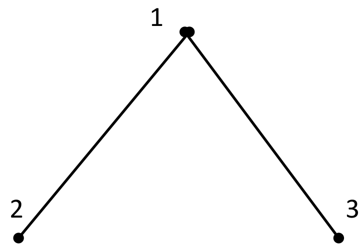
$L(K_{1,2})$



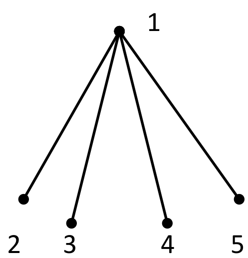
$K_{1,3}$:



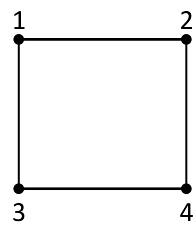
$L(K_{1,3})$



$K_{1,4}$:



$L(K_{1,4})$



From the above examples and from[7] we have the following observations

The line graph of P_n, $L(P_n)$ has

(i) n-1 vertices and n-2 edges

(ii) 2 vertices of degree 1 and n-3 vertices of degree 2.

Hence $L(P_n) = P_{n-1}$.

The line graph of C_n has

(i) n vertices and n edges

(ii) Each vertex is of degree 2

(iii) The graph is cyclic

Hence $L(C_n)= C_n$.

The line graph of $K_{1,n}$ has

(i) n-1 edges and n vertices

(ii) Each of the n vertices is of degree 2.

(iii) The graph is cyclic

$L(K_{1,n}) = C_n$ for $n \geq 3$.

Theorem 5.1

(1,2) - domination number of $L(P_n)$ is n-3.

Proof :

P_n has n vertices and n-1 edges and $L(P_n)$ is P_{n-1} with n-1 vertices and n-2 edges. Then by theorem 2.1, (1,2) - domination number of P_{n-1} with n-3. Hence (1,2) - domination in the line graph of $L(P_n)$ is n-3.

Theorem 5.2

$$(1,2)\text{ - domination number of } L(C_n) = \begin{cases} \frac{n}{2}, \text{if n is even} \\ n-2, \text{if n is odd} \end{cases}$$

Proof:

The line graph of C_n, $L(C_n)$ is C_n itself. So we can apply theorem 3.1

$$\text{Hence } \gamma_{(1,2)}(L(C_n)) = \begin{cases} \frac{n}{2}, \text{if } n \text{ is even} \\ n-2, \text{if } n \text{ is odd} \end{cases}$$

Theorem 5.3

(1,2) - domination number of $L(K_{1,n})$ is same as that of C_n.

Proof:

The line graph of $K_{1,n}$ is C_n.

Then by theorem 3.2,

$$\gamma_{(1,2)}(L(K_{1,n})) = \begin{cases} \frac{n}{2}, \text{if } n \text{ is even} \\ n-2, \text{if } n \text{ is odd} \end{cases}$$

6. Conclusions

In this paper the (1,2)-dominating sets and (1.2) - domination numbers of some standard graphs P_n, C_n and $K_{1,n}$have been discussed and the results extended to the line graphs of those graphs. It is also found that (1,2) - domination number of C_n and $K_{1,n}$ are same.

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