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Solitary Wave Solutions of the Modified Sasa - Satsuma Nonlinear Partial Differential Equation

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Abstract In this paper, we propose an easy and efficient way to analytically construct the solitary wave solutions of the modified Sasa-Satsuma equation. This approach called Bogning-DjeumenTchaho-Kofané method is based on the good management of the properties of the hyperbolic functions. First, we consider a shape of solution to construct as a combination of the functions of type solitary wave whose coefficients must be determined according to the parameter of the studied system. Thereafter, we obtain the equations of ranges of coefficients whose resolutions allow determining the values of the coefficients and in occurrence the solutions of the nonlinear partial differential equation.

Keywords Sasa-Satsuma Equation, BDK Method, Soliton, Nonlinear, Differential Equation

1. Introduction

The dynamics of physical systems is in general described by nonlinear partial differential equations (NPDE). If the increase of non-linearity on the other hand gives supplementary information in the understanding of the system, it complicates the analytic resolution of these equations. These NPDEs are seen in Mechanics of continuous media, in fluid mechanics, in nonlinear optic, in thermodynamics, kinetic chemistry… If the obtaining of these equations is one's in a while easy, the resolution is not always easy and at times constituted a veritable challenge. It is in this light that for the past years, researchers have been working hard and also proposing solutions and methods of resolution. In case of dynamics of solitary waves, many efficient methods have been stated. We can mention among others the tanh-sech method[1 -3], the homogeneous balance method[4-7], the extended tanh method[8,9], the tanh-coth method[10], the exp-function method[11-15], the jacobi's elliptic function method[16,17], the F-expansion method [18-20]. Beyond a multitude of methods many results were published in order to ameliorate the above mention methods or to extend them to other forms of equations[21-28]. In this work, we will use the principle that consists in decomposing the equation that we want to construct the solutions under the shape [22-26]

$$\begin{aligned}&\sum_{i,j,n} F\left(a_{ij}\right)/\cosh^n \alpha x+\sum_{i,j,m} G\left(a_{ij}\right)\sinh\alpha x/\cosh^m \alpha x\\&+\sum_{i,j,k} H\left(a_{ij}\right)\cosh^k \alpha x+\sum_{i,j,l} T\left(a_{ij}\right)\cosh^l \alpha x\sinh\alpha x\\&+\sum_{i,j} W\left(a_{ij}\right)=0,\end{aligned} \tag{1}$$

where $F\left(a_{ij}\right)$, $G\left(a_{ij}\right)$, $H\left(a_{ij}\right)$, $T\left(a_{ij}\right)$ and $W\left(a_{ij}\right)$ are functions of the coefficients a_{ij} to determine.

The paper is organized as follows: In section 2 , we look for the ranges of equations, in section 3, we solve the obtained ranges of equations and in section 4, we conclude the work.

2. Implementations of Equations of Coefficients

Sasa-Satsuma's equation is a well developed one of higher order obtained in certain order of nonlinearity. This equation is obtained principally in surface hydrodynamic wave paquets when perturbation development extends to the 4 order. We obtain similar equations in optics for waves of great velocity. The equation of Sasa-Satsuma presents itself in two forms [27, 28], the form that interest us is the modified one presented as follow

$$\begin{aligned}&iA_t+A_{xx}-2A^2A^*\\&+iv\left[A_{xxx}-6AA^*A_x-3A\left(AA^*\right)_x\right]=0,\end{aligned} \tag{2}$$

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where A_t represents the first derivative of the envelope A with respect to time, A_x represents the first derivative of the envelope A with respect to variable x and A^* stands for a conjugate complex of A. We propose to construct a solution made up of a combination of analytic forms of solitary waves. Not knowing the exact form capable of producing good results, we opt for the construction of the solution of the form

$$A(x,t)=\begin{bmatrix} a\operatorname{sec}h\alpha x+b\tanh\alpha x+c\operatorname{sec}h^2\alpha x \\ +d\sinh\alpha x\operatorname{sec}h^2\alpha x+... \end{bmatrix}\exp i\alpha t \quad (3)$$

where a, b, c, d and α can be real or complex. Given that the choice of the exact solution form of equation (2) is not always easy, we suppose that all the coefficients are complexes such that $a=a_r+ia_i$, $b=b_r+ib_i$, $c=c_r+ic_i$, $d=d_r+id_i$ and where a_r, a_i, b_r, b_i, c_r, c_i, d_r, d_i and α are real constants and $i^2=-1$. The choice of the solution in this form permits to have maximum possibilities as regard to the choice of the form of equation (3) which verifies best equation (2). Considering this, the introduction of equation (3) in equation (2) gives

$$\begin{aligned}&\sum_{n=1}^{7}F_n\left(a_r,a_i,b_r,b_i,c_r,c_i,d_r,d_i\right)\operatorname{sec}h^n\alpha x+\sum_{n=1}^{7}F'_n\left(a_r,a_i,b_r,b_i,c_r,c_i,d_r,d_i\right)\sinh\alpha x\operatorname{sec}h^n\alpha x\\&+i\sum_{n=1}^{7}G_n\left(a_r,a_i,b_r,b_i,c_r,c_i,d_r,d_i\right)\operatorname{sec}h^n\alpha x+i\sum_{n=1}^{7}G'_n\left(a_r,a_i,b_r,b_i,c_r,c_i,d_r,d_i\right)\sinh\alpha x\operatorname{sec}h^n\alpha x=0,\end{aligned} \quad (4)$$

where $F_n\left(a_r,a_i,b_r,b_i,c_r,c_i,d_r,d_i\right)$, $F'_n\left(a_r,a_i,b_r,b_i,c_r,c_i,d_r,d_i\right)$, $G_n\left(a_r,a_i,b_r,b_i,c_r,c_i,d_r,d_i\right)$, and $G'_n\left(a_r,a_i,b_r,b_i,c_r,c_i,d_r,d_i\right)$ are functions of the coefficients a_r, a_i, b_r, b_i, c_r, c_i, d_r, d_i to determine. Equation (4) is called the range equations of coefficients with factors $\sinh^m\alpha x\operatorname{sec}h^n\alpha x$ where $m=0,1$ and $n=1,2,...,7$. This equation has a real part and the imaginary part. On identifying the two parts of equation (4) equal to zero, we obtain the following equations classified in order of priority [24,25]. Hence the real part of equation (4) gives $F_n\left(a_r,a_i,b_r,b_i,c_r,c_i,d_r,d_i\right)=0$ as:

Term in $\operatorname{sec}h^7\alpha x$,

$$12\nu\alpha\left[d_i\left(c_r^2+c_i^2\right)+d_i\left(d_r^2+d_i^2\right)\right]-12\nu\alpha d_i\left(d_r^2+d_i^2\right)+12\nu\alpha d_i\left(c_r^2-c_i^2\right)-24\nu\alpha d_r c_r c_i=0, \quad (5)$$

Term in $\operatorname{sec}h^6\alpha x$,

$$\begin{aligned}&3\nu\alpha\begin{bmatrix}14a_ic_rd_r+14a_ic_id_i+7b_i\left(c_r^2+c_i^2\right)-4b_i\left(d_r^2+d_i^2\right)+6c_rc_ib_r-3c_r^2b_i+3c_i^2b_i\\-12c_ra_ia_r+6c_id_ra_r+6c_id_ia_i-3b_i\left(b_r^2+b_i^2\right)+6d_rd_ib_r-3d_r^2b_i+3b_id_i^2\end{bmatrix}\\&+18\nu\alpha d_i\left(a_rc_r+a_ic_i\right)+18\nu\alpha d_r\left(a_ic_r-a_rc_i\right)\\&+24\nu\alpha d_i\left(c_ra_r+c_ia_i\right)+24\nu\alpha d_r\left(c_ia_r-c_ra_i\right)+6\nu\alpha b_i\left(3c_r^2+3c_i^2-3d_r^2-3d_i^2\right)\\&+12\nu\alpha b_rd_rd_i-6\nu\alpha b_i\left(d_r^2-d_i^2\right)+18\nu\alpha c_i\left(a_rd_r+a_id_i\right)+18\nu\alpha c_r\left(a_id_r-a_rd_i\right)\\&-12\nu\alpha b_i\left(c_r^2-c_i^2\right)+24\nu\alpha b_rc_rc_i+2c_r\left(c_r^2+c_i^2\right)-4c_r\left(d_r^2+d_i^2\right)-2c_r\left(d_r^2-d_i^2\right)+4d_id_rc_i=0,\end{aligned} \quad (6)$$

Term in $\operatorname{sec}h^5\alpha x$,

$$3\nu\alpha\begin{bmatrix}-3a_r^2d_i+6a_ra_id_r+3a_i^2d_i-6a_rb_rc_i+6a_rb_ic_r+6a_ib_rc_r+6a_ib_ic_i-5a_rc_rb_i+5a_rc_ib_r+5a_ic_rb_r\\+5a_ic_ib_i+5d_i\left(a_r^2+a_i^2\right)-5a_ib_rc_r+5b_rc_ia_r+5b_ic_ra_r+5b_ic_ia_i+3b_r^2d_i-6b_rb_id_r-3b_i^2d_i\\+d_i\left(b_r^2+b_i^2\right)+3c_r^2d_i-6c_rc_id_r-3c_i^2d_i-7d_i\left(c_r^2+c_i^2\right)+2d_i\left(d_i^2+d_r^2\right)+4\left(d_r^2+d_i^2\right)d_i\end{bmatrix}$$

$$
\begin{aligned}
&+6\nu\alpha d_i\left[a_r^2+a_i^2-3\left(b_r^2+b_i^2\right)+3\left(d_r^2+d_i^2\right)-3\left(c_r^2+c_i^2\right)\right]\\
&+18\nu\alpha c_r\left(a_r b_i-a_i b_r\right)-18\nu\alpha c_i\left(a_i b_i+a_r b_r\right)+6\nu\alpha b_r\left(c_r a_i-c_i a_r\right)-6\nu\alpha b_i\left(c_i a_i+c_r a_r\right)\\
&+6\nu\alpha d_i\left(b_r^2-b_i^2\right)-12\nu\alpha d_r b_r b_i+6\nu\alpha d_i\left(a_r^2-a_i^2\right)-12\nu\alpha d_r a_i a_r+12\nu\alpha d_i\left(c_r^2-c_i^2\right)-24\nu\alpha d_r c_r c_i\\
&+24\nu\alpha^3 d_i+4\left(c_r^2+c_i^2\right)a_r-4\left(d_r^2+d_i^2\right)a_r+2\left(c_r^2-c_i^2\right)a_r+4c_r c_i a_i-2\left(d_r^2-d_i^2\right)a_r\\
&-4d_i d_r a_i-6\left(c_r d_r-c_i d_i\right)b_r-6\left(c_i d_r+c_r d_i\right)b_i-4\left(b_r c_r-b_i c_i\right)d_r-4\left(b_r c_i+b_i c_r\right)d_i\\
&-4\left(d_r b_r-d_i b_i\right)c_r-4\left(d_i b_r+b_i d_r\right)c_i=0,
\end{aligned}
\tag{7}
$$

Term in $\sec h^4\alpha x$,

$$
\begin{aligned}
&3\nu\alpha\begin{bmatrix}4a_r a_i b_r-2b_i a_r^2+2b_i a_i^2+4b_i\left(a_r^2+a_i^2\right)-5a_r c_i d_r-5a_i c_r d_r+5a_r c_r d_i-5a_i c_i d_i-6a_r d_i c_r\\
-6a_i d_r c_r+6a_r d_r c_i-6a_i d_i c_i-2b_i\left(b_r^2+b_i^2\right)+10b_i\left(d_r^2+d_i^2\right)-6b_i\left(c_r^2+c_i^2\right)-4c_r c_i b_r\\
+2c_r^2 b_i-2b_i c_i^2-5c_r d_i a_r-5c_i d_r a_r+5c_r d_r a_i-5c_i d_i a_i-4d_r d_i b_r+2b_i d_r^2-2b_i d_i^2\end{bmatrix}\\
&-6\nu\alpha\left(b_r^2+b_i^2\right)b_i+24\nu\alpha\left(d_r^2+d_i^2\right)b_i\\
&-12\nu\alpha\left(c_r^2+c_i^2\right)b_i-18\nu\alpha b_i d_r^2+18\nu\alpha b_i d_i^2+36\nu\alpha d_i d_r b_r+12\nu\alpha d_r a_r c_i\\
&-12\nu\alpha d_i a_i c_i-12\nu\alpha d_i a_r c_r-12\nu\alpha d_r a_i c_r-18\nu\alpha d_i c_r a_r-18\nu\alpha d_r c_i a_r-18\nu\alpha d_i c_i a_i\\
&+18\nu\alpha d_r c_r a_i-18\nu\alpha c_i a_r d_r-18\nu\alpha c_r a_i d_r+18\nu\alpha c_r a_r d_i-18\nu\alpha c_i a_i d_i-24\nu\alpha c_r c_i b_r\\
&+12\nu\alpha\left(c_r^2-c_i^2\right)b_i+6\nu\alpha^3 b_i+2\left(a_r^2-a_i^2\right)c_r+4a_r a_i c_i-2\left(b_r^2-b_i^2\right)c_r\\
&-4b_r b_i c_i+2\left(d_r^2-d_i^2\right)c_r+4d_r d_i c_i+4\left(a_r^2+a_i^2\right)c_r-4\left(b_r^2+b_i^2\right)c_r+2\left(d_r^2+d_i^2\right)c_r-4\left(a_r b_r-a_i b_i\right)d_r\\
&-4\left(a_r b_i+a_i b_r\right)d_i-4\left(a_r d_r-a_i d_i\right)b_r-4\left(a_r d_i+a_i d_r\right)b_i-4\left(d_r b_r-d_i b_i\right)a_r-4\left(d_i b_r+d_r b_i\right)a_i-6\alpha^2 c_r=0,
\end{aligned}
\tag{8}
$$

Term in $\sec h^3\alpha x$,

$$
\begin{aligned}
&3\nu\alpha\begin{bmatrix}-4a_r a_i d_r+2a_r^2 d_i-2d_i a_i^2-5a_r b_i c_r-5a_i b_r c_r+5c_i a_r b_r-5a_i b_i c_i-3a_r c_i b_r-3a_i c_r b_r+3a_r c_r b_i-3a_i c_i b_i\\
-4d_i\left(a_r^2+a_i^2\right)+8b_r b_i d_r-4d_i b_r^2+4b_i^2 d_i-4c_r b_i a_r-4c_i b_r a_r+4a_i c_r b_r-4a_i c_i b_i-2d_i\left(d_r^2+d_i^2\right)\end{bmatrix}\\
&+24\nu\alpha\left(b_r^2+b_i^2\right)d_i-12\nu\alpha\left(a_r^2+a_i^2\right)d_i\\
&+12\nu\alpha b_r b_i d_r-6\nu\alpha\left(b_r^2-b_i^2\right)d_i-6\nu\alpha\left(a_r^2+a_i^2\right)b_i-6\nu\alpha\left(d_r^2+d_i^2\right)b_i\\
&+18\nu\alpha c_r a_r b_i-18\nu\alpha c_i a_i b_i-18\alpha\nu c_i a_r b_r-18\nu\alpha c_r a_i b_r-12\nu\alpha b_i c_r a_r-12\nu\alpha b_r c_i a_r\\
&+12\nu\alpha b_r c_r a_i-12\nu\alpha b_i c_i a_i+12\nu\alpha a_i a_r d_r-6\nu\alpha\left(a_r^2-a_i^2\right)d_i-12\nu\alpha a_i a_r b_r\\
&+6\nu\alpha\left(a_r^2-a_i^2\right)b_i+6\nu\alpha b_i a_r c_r+6\nu\alpha b_r a_i c_r-6\nu\alpha b_r a_r c_i+6\nu\alpha b_i a_i c_i-2\alpha^2 a_r\\
&+2a_r\left(a_r^2+a_i^2\right)-4a_r\left(b_r^2+b_i^2\right)-4a_r\left(d_r^2+d_i^2\right)-2a_r\left(b_r^2-b_i^2\right)-4b_r b_i a_i+2\left(d_r^2-d_i^2\right)a_r\\
&+4d_i d_r a_i+4\left(b_r c_r-b_i c_i\right)d_r+4\left(b_r c_i+b_i c_r\right)d_r+4\left(d_r b_r-d_i b_i\right)c_r+4\left(d_i b_r+b_i d_r\right)c_i\\
&+4\left(c_r d_r-c_i d_i\right)b_r+4\left(c_i d_r+c_r d_i\right)b_i=0.
\end{aligned}
\tag{9}
$$

The imaginary part of equation (4) leads to the following equations $G_n\left(a_r,a_i,b_r,b_i,c_r,c_i,d_r,d_i\right)=0$ such as:

Term in $\sec h^7\alpha x$,

$$
\begin{aligned}
&-3\nu\alpha\left[4d_r\left(c_r^2+c_i^2\right)-4\left(d_r^2+d_i^2\right)d_r\right]+12\nu\alpha d_r\left(d_r^2+d_i^2\right)+12\nu\alpha\left(c_r^2-c_i^2\right)d_r\\
&+2\nu\alpha d_i c_r c_i=0,
\end{aligned}
\tag{10}
$$

Term in $\sec h^6 \alpha x$,

$$\begin{aligned}
&-3\nu\alpha\begin{bmatrix} 7a_rc_rd_r - 7a_ic_id_r + 7a_rc_id_i + 7a_ic_rd_i + 7a_rd_rc_r - 7a_id_ic_r + 7a_rd_ic_i + 7a_id_rc_i \\ +7b_r\left(c_r^2+c_i^2\right) - 4b_r\left(d_r^2+d_i^2\right) + 3c_r^2b_r - 3c_i^2b_r + 6c_rc_ib_i + 6c_ra_r^2 - 6c_id_ia_r \\ +6c_ra_i^2 + 6c_id_ra_i - 3b_r\left(b_r^2+b_i^2\right) + 3d_r^2b_r - 3d_i^2b_r + 6d_rd_ib_i \end{bmatrix} \\
&-18\nu\alpha d_r\left(a_rc_r + a_ic_i\right) - 18\nu\alpha d_i\left(a_rc_i - a_ic_r\right) \\
&-24\nu\alpha d_r\left(c_ra_r + a_ic_i\right) - 24\nu\alpha d_i\left(c_ra_i - c_ia_r\right) - 6\nu\alpha b_r\left[3\left(c_r^2+c_i^2\right) - 3\left(d_r^2+d_i^2\right)\right] \\
&-6\nu\alpha b_r\left(d_r^2 - d_i^2\right) + 12\nu\alpha b_id_rd_i - 18\nu\alpha c_r\left(a_rd_r + a_id_i\right) - 18\nu\alpha c_i\left(a_rd_i - a_id_r\right) \\
&-12\nu\alpha b_r\left(c_r^2 - c_i^2\right) - 24\nu\alpha b_ic_rc_i + 2\left(c_r^2+c_i^2\right)c_i - 4\left(d_r^2+d_i^2\right)c_i \\
&-2\left(d_r^2 - d_i^2\right)c_i - 4d_rd_ic_r = 0,
\end{aligned} \tag{11}$$

Term in $\sec h^5 \alpha x$,

$$\begin{aligned}
&-3\nu\alpha\begin{bmatrix} 3a_r^2d_r + 6a_ra_id_i - 3a_i^2d_r + 6a_rb_rc_r + 6a_rb_ic_i + 6a_ib_rc_i - 6a_ib_ic_r + 5a_rc_rb_r + 5a_rc_ib_i + 5a_ic_rb_i \\ -5a_ic_ib_r + 5d_r\left(a_r^2+a_i^2\right) + 5b_rc_ra_r + 5b_rc_ia_i + 5b_ic_ra_i - 5b_ic_ia_r - 3b_r^2d_r - 6b_rb_id_i + 3b_i^2d_r \\ +d_r\left(b_r^2+b_i^2\right) - 3c_r^2d_r - 6c_rc_id_i + 3c_i^2d_r + 4\left(d_r^2+d_i^2\right)d_r - 7d_r\left(c_r^2+c_i^2\right) + 2d_r\left(d_r^2+d_i^2\right) \end{bmatrix} \\
&-6\nu\alpha d_r\left(a_r^2 + a_i^2 - 3b_r^2 - 3b_i^2 - 3d_r^2 - 3d_i^2 - 3c_r^2 - 3c_i^2\right) \\
&-18\nu\alpha c_i\left(a_ib_r - a_rb_i\right) + 18\nu\alpha c_r\left(a_rb_r + a_ib_i\right) - 6\nu\alpha b_i\left(c_ia_r - c_ra_i\right) \\
&+6\nu\alpha b_r\left(c_ra_r + c_ia_i\right) + 6\nu\alpha d_r\left(b_r^2 - b_i^2\right) + 12\nu\alpha d_ib_rb_i \\
&+6\nu\alpha d_r\left(c_ra_r + c_ia_i\right) + 6\nu\alpha d_r\left(b_r^2 - b_i^2\right) + 12\nu\alpha d_ib_rb_i + 6\nu\alpha d_r\left(a_r^2 - a_i^2\right) + 12\nu\alpha d_ia_ia_r \\
&+12\nu\alpha d_r\left(c_r^2 - c_i^2\right) + 24\nu\alpha d_ic_ic_r - 24\nu\alpha^3d_r + 4\left(c_r^2+c_i^2\right)a_i - 4\left(d_r^2+d_i^2\right)a_i \\
&+4c_ic_ra_r - 2\left(c_r^2 - c_i^2\right)a_i + 2\left(d_r^2 - d_i^2\right)a_i - 4d_id_ra_r + 6c_rd_rb_i - 6c_id_ib_i - 6\left(c_id_r + c_rd_i\right)b_r \\
&+4\left(b_rc_r - b_ic_i\right)d_i - 4\left(b_rc_i + b_ic_r\right)d_r + 4\left(d_rb_r - d_ib_i\right)c_i - 4\left(d_ib_r + b_id_r\right)c_r = 0,
\end{aligned} \tag{12}$$

Term in $\sec h^4 \alpha x$,

$$\begin{aligned}
&-3\nu\alpha\begin{bmatrix} 2a_r^2b_r - 2a_i^2b_r + 4a_ra_ib_i + 4b_r\left(a_r^2+a_i^2\right) - 5a_rc_rd_r + 5a_ic_id_r - 5a_rc_id_i - 5a_ic_rd_i - 6a_rd_rc_r \\ +6a_id_ic_r - 6a_rd_ic_i - 6a_id_rc_i - 2b_r\left(b_r^2+b_i^2\right) - 10b_r\left(d_r^2+d_i^2\right) - 6b_r\left(c_r^2+c_i^2\right) - 2c_r^2b_r \\ +2c_i^2b_r - 4c_rc_ib_i - 5c_rd_ra_r + 5c_id_ia_r - 5c_rd_ia_i - 5c_id_ra_i - 2b_rd_r^2 + 2b_rd_i^2 - 4d_rd_ib_i \end{bmatrix} \\
&-6\nu\alpha b_r\left(a_r^2+a_i^2\right) + 6\nu\alpha b_r\left(b_r^2+b_i^2\right) \\
&-24\nu\alpha b_r\left(d_r^2+d_i^2\right) + 12\nu\alpha b_r\left(c_r^2+c_i^2\right) - 18\nu\alpha b_rd_r^2 + 18\nu\alpha b_rd_i^2 - 36\nu\alpha b_id_rd_i \\
&-12\nu\alpha d_ia_ic_r + 12\nu\alpha d_ra_rc_r + 12\nu\alpha d_ia_rc_i + 12\nu\alpha d_ra_ic_i - 18\nu\alpha d_ic_ia_i + 18\nu\alpha d_rc_ra_r \\
&+18\nu\alpha d_ic_ra_i + 18\nu\alpha d_rc_ia_i - 18\nu\alpha c_ia_id_r + 18\nu\alpha c_ra_rd_r + 18\nu\alpha c_ia_rd_i + 18\nu\alpha c_ra_id_i \\
&+12\nu\alpha\left(c_r^2 - c_i^2\right)b_r + 24\nu\alpha c_rc_ib_i - 6\nu\alpha^3b_r + 4a_ra_ic_r - 2\left(a_r^2 - a_i^2\right)c_i + 2\left(b_r^2 - b_i^2\right)c_i \\
&+2\left(b_r^2 - b_i^2\right)c_i - 4b_rb_ic_r + 4d_rd_ic_r - 2\left(d_r^2 - d_i^2\right)c_i + 4\left(a_r^2+a_i^2\right)c_i - 4\left(b_r^2+b_i^2\right)c_i \\
&-4\left(d_r^2+d_i^2\right)c_i + 4\left(a_rb_r - a_ib_i\right)d_i - 4\left(a_rb_i + a_ib_r\right)d_r + 4\left(a_rd_r - a_id_i\right)b_i - 4\left(a_rd_i + a_id_r\right)b_r \\
&+4\left(d_rb_r - b_id_i\right)a_i - 4\left(d_ib_r + b_id_r\right)a_r - 6\alpha^2c_i = 0,
\end{aligned} \tag{13}$$

Term in $\sec h^3\alpha x$,

$$-3\nu\alpha\begin{bmatrix}-2a_r^2d_r+4a_i^2d_r-4d_ia_ra_i-5a_rb_rc_r+5a_ib_ic_r-5a_rb_ic_i-5a_ib_rc_i-3a_rc_rb_r+3a_ic_ib_r-3a_rc_ib_i-3a_ic_rb_i\\-4d_r\left(a_r^2+a_i^2\right)+4b_r^2d_r-4b_i^2d_r+8b_rb_id_i-4c_ra_rb_r+4a_rc_ib_i-4a_ic_rb_i-4a_ic_ib_r-2d_r\left(d_r^2+d_i^2\right)\end{bmatrix}$$

$$\begin{aligned}&-24\nu\alpha\left(b_r^2+b_i^2\right)d_r+12\nu\alpha\left(a_r^2+a_i^2\right)d_r\\&-6\nu\alpha\left(b_r^2-b_i^2\right)d_r-12\nu\alpha b_rb_id_i+6\nu\alpha\left(a_r^2+a_i^2\right)b_r+6\nu\alpha\left(d_r^2+d_i^2\right)b_r\\&+18\nu\alpha c_ra_rb_r-18\nu\alpha c_ia_ib_r+18\nu\alpha c_ia_rb_i+18\nu\alpha c_ra_ib_i+12\nu\alpha b_rc_ra_r-12\nu\alpha b_ic_ia_r\\&+12\nu\alpha b_ic_ra_i+12\nu\alpha b_rc_ia_i+6\nu\alpha\left(a_r^2-a_i^2\right)d_r\\&+12\nu\alpha a_ia_rd_i+6\nu\alpha b_ra_rc_r-6\nu\alpha b_ia_ic_r+6\nu\alpha b_ia_rc_i+6\nu\alpha b_ra_ic_i\\&+6\nu\alpha\left(a_r^2-a_i^2\right)b_r+12\nu\alpha a_ia_rb_i-2\alpha^2a_i=0.\end{aligned}\tag{14}$$

As mentioned above, the choice of the form of solution when we use the BDK method [34,35] is not always easy. In the setting of this work, we wanted in a first time that the coefficients a, b, c and d are complex ($a=a_r+ia_i$, $b=b_r+ib_i$, $c=c_r+ic_i$); this in the goal to multiply the possibilities of obtaining the shape of the most suitable solution. This being, the sets of equations (10), (11), (12), (13) and (14) possess 8 unknowns a_r, a_i, b_r, b_i, c_r, c_i, d_r and d_i whose resolution is not easy because of their nonlinearity. Of all considered hypotheses, we got two that allowed us to get acceptable solutions.

The groups of equations (10), (11), (12), (13) and (14) lead to identities while the groups of equations (5), …, (9) become

Term in $\sec h^7\alpha x$,

$$\alpha d_ic_r^2=0,\tag{15}$$

Term in $\sec h^6\alpha x$,

$$-24\nu\alpha b_ic_r^2-15\nu\alpha b_id_i^2-9\nu\alpha b_i^3+2c_r^3-6c_rd_i^2=0,\tag{16}$$

Term in $\sec h^5\alpha x$,

$$\begin{aligned}&18\nu\alpha d_ia_r^2+30\nu\alpha a_rc_rb_i-30\nu\alpha d_ib_i^2-18\nu\alpha d_ic_r^2+36\nu\alpha d_i^3+24\nu\alpha_r^3d_i\\&+6a_rc_r^2-2a_rd_i^2-6c_rd_ib_i=0\end{aligned}\tag{17}$$

Term in $\sec h^4\alpha x$,

$$\begin{aligned}&6\nu\alpha b_ia_r^2-30\nu\alpha c_ra_rd_i-12\nu\alpha b_i^3+66\nu\alpha b_id_i^2-12\nu\alpha b_ic_r^2+6\nu\alpha^3b_i\\&+6c_ra_r^2-2c_rb_i^2-8a_rd_ib_i-6\alpha^2c_r=0,\end{aligned}\tag{18}$$

Term in $\sec h^3\alpha x$,

$$\begin{aligned}&-12\nu\alpha d_ia_r^2+30\nu\alpha a_rb_ic_r+6\nu\alpha d_i^3\\&+30\nu\alpha d_ib_i^2-6\nu\alpha b_id_i^2-2\alpha^2a_r\\&+2a_r^3-2a_rb_i^2-6a_rd_i^2=0.\end{aligned}\tag{19}$$

3. Resolution of the Range Equations

The equation (15) imposes us three possibilities, the case where $d_i=0$ and $c_r=0$, the case where $d_i\neq0$ and $c_r=0$, and the case where $d_i=0$ and $c_r\neq0$. But in the continuation we are going to be interested in the last two quoted cases.

First case

When $d_i=0$ and $c_r\neq0$, the equations (16), (17),..., (19) become respectively

$$24\nu\alpha b_ic_r^2+2c_r^3-9\nu\alpha b_i^3=0,\tag{20}$$

$$5\nu\alpha b_i+c_r=0,\tag{21}$$

$$\begin{aligned}&3\nu\alpha b_ia_r^2-6\nu\alpha b_i^3-6\nu\alpha b_ic_r^2\\&+3\nu\alpha^3b_i+3c_ra_r^2-c_rb_i^2-3\alpha^2c_r=0\end{aligned},\tag{22}$$

$$15\nu\alpha b_ic_r-\alpha^2+a_r^2-b_i^2=0.\tag{23}$$

Taking into account of equation (21) in equation (20) gives us the constraint $\nu\alpha=\pm\sqrt{9/350}$ and thereafter, the combination of equations (20), (22) and (23) yields

$$b_i=\pm\alpha\sqrt{3/20},\tag{24}$$

$$c_r=\pm15\alpha\sqrt{21000}/21000,\tag{25}$$

$$a_r = \pm\alpha\sqrt{121/70}. \tag{26}$$

So when we consider equations (24), (25) and (26) in equation (3) we obtain the solution

$$A(x,t) = \left[\pm\alpha\sqrt{\frac{121}{70}}\sec h\alpha x \pm i\alpha\sqrt{\frac{3}{20}}\tanh\alpha x \pm \frac{\alpha\sqrt{21000}}{1400}\sec h^2\alpha x\right]\exp i\alpha t, \tag{27}$$

While observing the solution equations (11), we note that we should choose the constants of the initial solution (3) as they verify the following criteria $\alpha = cste$, $a_r \neq 0$, $b_i \neq 0$, $c_r \neq 0$ and $a_i = 0 = b_r = c_i = d_r = d_i = 0$.

Second case: $\alpha \neq 0$, $a_r \neq 0$, $c_r \neq 0$, $d_i \neq 0$, $b_r \neq 0$ and $a_i = b_i = c_i = d_r = 0.$

The gotten below equations essentially come from the real part of equation (4) because its imaginary part produced merely identities. Thus, it follows:

Term in $\sec h^7\alpha x$,

$$12\nu\alpha d_i c_r^2 = 0, \tag{28}$$

Term in $\sec h^6\alpha x$,

$$12\nu a_r d_i + c_r^2 - d_i^2 = 0, \tag{29}$$

Term in $\sec h^5\alpha x$,

$$\begin{aligned}&18\nu\alpha a_r^2 d_i - 18\nu\alpha d_i c_r^2 + 36\nu\alpha d_i^3\\&+24\nu\alpha^3 d_i + 6a_r c_r^2 - 2a_r d_i^2 = 0,\end{aligned} \tag{30}$$

Term in $\sec h^4\alpha x$,

$$-30\nu\alpha a_r d_i + 6a_r^2 - 6b_r^2 - 6\alpha^2 = 0. \tag{31}$$

From equation (28) we consider the case where $d_i \neq 0$ and $c_r = 0$. Hence, the equations (29), (30) and (31) respectively write

$$12\nu\alpha a_r - d_i = 0, \tag{32}$$

$$9\nu\alpha a_r^2 + 18\nu\alpha d_i^2 + 12\nu\alpha^3 - a_r d_i = 0, \tag{33}$$

$$-5\nu\alpha a_r d_i + a_r^2 - b_r^2 - \alpha^2 = 0. \tag{34}$$

Solving the equations (32), (33) and (34) we obtain

$$a_r = \pm\sqrt{4\alpha^2/\left(4-864\nu^2\alpha^2\right)}, \tag{35}$$

$$d_i = \pm 12\nu\alpha\sqrt{4\alpha^2/\left(4-864\nu^2\alpha^2\right)}, \tag{36}$$

$$b_r = \pm\alpha\sqrt{624\nu^2\alpha^2/\left(4-864\nu^2\alpha^2\right)}, \tag{37}$$

with $-2/\sqrt{864} \prec \nu\alpha \prec 2/\sqrt{864}$. Substituting equations (35), (36) and (37) in equation (3) we obtain the solution

$$A(x,t) = \begin{bmatrix}\pm\sqrt{4\alpha^2/\left(4-864\nu^2\alpha^2\right)}\sec h\alpha x\\ \pm\alpha\sqrt{624\nu^2\alpha^2/\left(4-864\nu^2\alpha^2\right)}\tanh\alpha x\\ \pm 12i\nu\alpha\sqrt{4\alpha^2/\left(4-864\nu^2\alpha^2\right)}\sinh\alpha x\sec h^2\alpha x\end{bmatrix}\exp i\alpha t \tag{38}$$

According to the solution (38), one realizes that one should have merely chosen to construct the solution of the equation (2) under the shape of equation (3) as the following conditions are verified: $\alpha \neq 0$, $a_r \neq 0$, $a_i \neq 0$, $b_r \neq 0$ and $a_i = b_i = c_i = c_r = d_r = 0$.

4. Conclusions

The Sasa-Satsuma equation under its modified shape as considered in this work is not always easy to integrate. Not knowing initially the shape of solution which is susceptible to verify this equation, we proposed to construct a solution that is the combination of the hyperbolic functions of type solitary wave. The use of the BDK method allowed us to obtain successfully the equations of range of coefficients that allowed assigning some values to these coefficients. These values of determined coefficients also permit to give some particular solutions. We note that beyond the calculations that require a lot of concentrations, the obtained solutions confirm the fact that Sasa-Satsuma equation is integrable. It is sufficient to really choose the shape of solution to construct. Our satisfaction is especially due to the fact that the obtaining of these solutions was possible thanks to the choice of the method that we had used. This method is very adapted to the greatly nonlinear partial differential equation which present the scattering terms.

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