

Non-linear Thickness Variation on Harmonically Thermal Induced Vibration of a Rectangular Plate

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Abstract An analysis and numerical results are presented for free transverse vibrations of rectangular plates of non-linear thickness variation and harmonically temperature distribution on the basis of classical plate theory. Following Levy approach i.e. the two parallel edges are simply supported, the fourth-order differential equation governing the motion of such plates of non-linear varying thickness in one direction with harmonic temperature distribution, has been solved by using the quintic splines interpolation technique for two different combinations of clamped and simply supported boundary conditions at the other two edges. An algorithm for computing the solution of this differential equation is presented, for the case of equal intervals. Effect of the taper constants and thermal constant together with other plate parameters such as aspect ratio on the natural frequencies of vibration is illustrated for the first three modes of vibration.

Keywords Non-Linear, Thickness Variation, Harmonic Temperature, Vibration, Rectangular Plate

1. Introduction

In the recent past, there was a phenomenal increase in the development of elastic materials due to the desirability of lightweight, high strength, corrosion resistance and high-temperature performance requirements in modern technology. Plates of composite materials are widely used in many engineering structures and machines. In this era of science and technology plates of various shapes and of variable thickness may be regarded as a first approximation to wings and blades and occur as panels in many forms of engineering structures. Thus knowledge of their natural frequencies is of considerable importance at the design stage in order to avoid resonance excited by internal or external forces. Thus, their design requires an accurate determination of their natural frequencies and mode shapes. An extensive review of the work up to 1985 on linear vibration of isotropic/anisotropic plates of various geometries has been given by Leissa in his monograph [1]. The studies on vibration of rectangular plates with uniform/non-uniform thickness with various edge conditions after 1985 has been carried out by a number of researchers and are reported in refs.[2-5].

Jain and Soni [6] find out the free vibrations of rectangular plates of parabolically varying thickness. Tomar *et al.*[7] studied the free vibrations of an isotropic non-homogeneous infinite plate of parabolically varying thickness. Singh and

Jain[8] have studied the free asymmetric transverse vibration of parabolically varying thickness of polar orthotropic annular plate with flexible edge conditions. Kumar and Tomar[9] studied the free transverse vibrations of monoclinic rectangular plates with continuously varying thickness and density. Gupta *et al.*[10] studied the vibration analysis of non-homogeneous circular plate of nonlinear thickness variation by differential quadrature method. Gupta *et al.*[11] have studied the thermal effect on vibration of non-homogeneous orthotropic rectangular plate having bi-directional parabolically varying thickness. Feng and Min[12] have studied the vibrations of axially moving visco-elastic plate with parabolically varying thickness. Gupta *et al.*[13] studied the vibration of visco-elastic parallelogram plate with parabolic thickness variation. Bhatnagar and Gupta[14] did the vibration analysis of visco-elastic circular plate subjected to thermal gradient. Gupta and Sharma[15] studied the effect of thermal gradient on vibration of trapezoidal plate of linearly varying thickness. Gupta and Singhal[16] studied the effect of non-homogeneity on thermally induced vibration of orthotropic visco-elastic rectangular plate of linearly varying thickness. Tomar and Gupta[17] studied the effect of thermal gradient on frequencies of an orthotropic rectangular plate whose thickness varies in two directions. Gupta and Khanna[18] solved the problem of visco-elastic rectangular plate with linearly thickness variations in both directions. Gupta and Sharma[19] studied the thermally induced vibration of orthotropic trapezoidal plate of linearly varying thickness. Gupta and Sharma[20] discussed the effect of linear thermal gradient on vibration of trapezoidal plates whose thickness varies parabolically. Recently, Gupta and

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Published online at <http://journal.sapub.org/ajcam>

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Sharma[21] study the thermally induced vibration of non-homogeneous trapezoidal plate with varying thickness and density.

In an up-to-date survey of literature, the authors have not come across any study dealing with vibration of non-linear rectangular plates with the exception of Gupta et al.[10], which deals non-homogeneous circular plate of non-linear thickness variation. Keeping this in view, a study dealing with transverse vibrations of rectangular plates of non-linear (combination of linear and quadratic variation) varying thickness along one direction and harmonic temperature distribution is presented employing classical plate theory. The governing differential equation for such plates with two opposite edges simply supported reduces to fourth-order differential equation with variable coefficients whose analytical solution is not feasible. Quintic splines interpolation technique has been employed to obtain the natural frequencies for two different combinations of clamped and simply supported boundary conditions at the other two edges. This method is preferred because a chain of lower-order approximations may yield a better accuracy than a global higher-order approximation[21] and natural boundary conditions can be considered easily. The effect of various plate parameters has been studied on the natural frequencies for the first three modes of vibration. The consideration of non-linearity thickness variation, harmonic temperature distribution and aspect ratio leads to a very complex problem involving several parameters. However, with the choice of Levy approach and one-dimensional variations in thickness and temperature distribution, one can find the approximate solution of the present problem. This type of variation in thickness and temperature distribution consideration is of interest since it provides reasonable

approximation to variation. Thus, the present study of theoretically investigated vibrational characteristics will be of interest to design engineers.

2. Analysis and Equation of Motion

It is assumed that the rectangular plate is subjected to a harmonic temperature distribution along the length i.e. in x-directions[23]:

$$T=T_0 \cos(\pi/2)X \quad (1)$$

where T denotes the temperature excess above the reference temperature at any point at distance $X = \frac{x}{a}$ and T_0 denotes the temperature excess above reference temperature at the end i.e. $x=a$ or $X=1$.

The temperature dependence of the modulus of elasticity for most of engineering materials is given by[24]:

$$E(T) = E_0(1 - \gamma T) \quad (2)$$

where E_0 is the value of the Young's modulus at reference temperature i.e. $T=0$ and γ is the slope of the variation of E with T .

Taking as the reference temperature, the temperature at the end of plate i.e. at $X=1$, the modulus variation in view of (1) and (2), becomes

$$E(X)=E_0[1 - \alpha \cos(\pi/2)X] \quad (3)$$

where $\alpha = \gamma T_0$ ($0 \leq \alpha < 1$), a constant known as temperature constant.

The differential equation governing the free transverse motion of an elastic rectangular plate of length a , breadth b , thickness h and density ρ is

$$D \nabla^4 w + 2 \frac{\partial D}{\partial x} \frac{\partial}{\partial x} \nabla^2 w + 2 \frac{\partial D}{\partial y} \frac{\partial}{\partial y} \nabla^2 w + \nabla^2 D \nabla^2 w + (v-1) \left\{ \frac{\partial^2 D}{\partial x^2} \frac{\partial^2 w}{\partial y^2} - 2 \frac{\partial^2 D}{\partial x \partial y} \frac{\partial^2 w}{\partial x \partial y} + \frac{\partial^2 D}{\partial y^2} \frac{\partial^2 w}{\partial x^2} \right\} + \rho h \frac{\partial^2 w}{\partial t^2} = 0. \quad (4)$$

where w is transverse displacement.

Let us assume that the two opposite edges $y=0$ and $y=b$ are simply supported and that thickness varies in x-direction only. Therefore, the thickness h and flexural rigidity D of the plate become a function of x only. For harmonic vibration, w may have Levy type solution as

$$w(x,y,t) = W_1(x) \sin(m\pi y/b) e^{ipt} \quad (5)$$

where p is the circular frequency and m is a positive integer.

Substitution of equation (5) in (4) gives

$$D W_{1,xxxx} + 2D_{1,x} W_{1,xxx} + [-2(m^2\pi^2/b^2)D + D_{1,xx}] W_{1,xx} + [-2(m^2\pi^2/b^2)D_{1,x}] W_{1,x} + [(m^4\pi^4/b^4)D - v(m^2\pi^2/b^2)D_{1,xx}] W_1 = \rho h p^2 W_1 \quad (6)$$

A comma followed by a suffix denotes partial differentiation with respect to that variable.

Thus equation (6) reduces to a form independent of y and on introducing the non-dimensional variables

$$H=h/a, W=W_1/a, X=x/a, D_1=D/a^3 \quad (7)$$

the differential equation (6) reduces to

$$D_1 W_{,xxxx} + 2D_{1,x} W_{,xxx} + [D_{1,xx} - 2r^2 D_1] W_{,xx} - 2r^2 D_{1,x} W_{,x} + r^2 [r^2 D_1 - v D_{1,xx}] W = \rho H a^2 p^2 W \quad (8)$$

where $r^2 = (m\pi a/b)^2$.

Since thickness varies non-linearly in x-direction only, therefore, one can assume

$$H=H_0(1+\beta_1 X+\beta_2 X^2) \quad (9)$$

where β_1 and β_2 are taper constants such that $|\beta_1| \leq 1$, $|\beta_2| \leq 1$ and $\beta_1+\beta_2 > -1$, H_0 is thickness at $X=0$.

Considering equation (3) and (9) with help of (7), the expression for rigidities D_1 comes out as

$$D_1 = D_0 [1 - \alpha \cos(\pi/2)X] (1+\beta_1 X+\beta_2 X^2)^3 \quad (10)$$

where $D_0 = E_0 H_0^3 / 12(1-v^2)$

Using equations (8) to (10), one obtains the equation of motion as:

$$[1 - \alpha \cos(\pi/2)X_q] (1 + \beta_1 X_q + \beta_2 X_q^2)^2 W_{xxxx} + 2[\alpha(\pi/2) \sin((\pi/2)X_q) (1 + \beta_1 X_q + \beta_2 X_q^2)^2 + 3(1 - \alpha \cos(\pi/2)X_q) (1 + \beta_1 X_q + \beta_2 X_q^2)(\beta_1 + 2\beta_2 X_q)] W_{xxx} + [(\alpha(\pi/2)^2 \cos((\pi/2)X_q) (1 + \beta_1 X_q + \beta_2 X_q^2)^2 + 6\alpha(\pi/2) \sin((\pi/2)X_q) (1 + \beta_1 X_q + \beta_2 X_q^2)(\beta_1 + 2\beta_2 X_q) + 6(1 - \alpha \cos(\pi/2)X_q) (\beta_1 + 2\beta_2 X_q)^2 + 6(1 - \alpha \cos(\pi/2)X_q) (\beta_1 + \beta_2 X_q^2) \beta_2] - 2r^2 (1 - \alpha \cos(\pi/2)X_q) (1 + \beta_1 X_q + \beta_2 X_q^2)^2] W_{xx} - 2r^2 [\alpha(\pi/2) \sin((\pi/2)X_q) (1 + \beta_1 X_q + \beta_2 X_q^2)^2 + 3(1 - \alpha \cos(\pi/2)X_q) (1 + \beta_1 X_q + \beta_2 X_q^2)(\beta_1 + 2\beta_2 X_q)] W_x + r^2 [r^2 (1 - \alpha \cos(\pi/2)X_q) (1 + \beta_1 X_q + \beta_2 X_q^2)^2 - v((\alpha(\pi/2)^2 \cos((\pi/2)X_q) (1 + \beta_1 X_q + \beta_2 X_q^2)^2 + 6\alpha(\pi/2) \sin((\pi/2)X_q) (1 + \beta_1 X_q + \beta_2 X_q^2)(\beta_1 + 2\beta_2 X_q) + 6(1 - \alpha \cos(\pi/2)X_q) (\beta_1 + 2\beta_2 X_q)^2 + 6(1 - \alpha \cos(\pi/2)X_q) (\beta_1 + \beta_2 X_q^2) \beta_2)] W = \lambda^2 W \quad (11)$$

where

$$\lambda^2 = (p^2 a^2 / (E_0 \rho)) (12(1 - v^2) / H_0^2) \quad (12)$$

is a frequency parameter.

3. Method of Solution

Let $f(X)$ be a function with continuous derivatives in the range $(0, l)$. Choose $(n + 1)$ points $X_0, X_1, X_2, \dots, X_n$ in the range $0 \leq X \leq l$ such that $0 = X_0 < X_1 < X_2 < X_3 < \dots < X_n = l$.

Let the approximating function $W(X)$ for $f(X)$ be a quintic spline with the following properties :

- (a) $W(X)$ is a quintic polynomial in each interval (X_k, X_{k+1}) ,
- (b) $W(X_k) = f(X_k)$, $k = 0(1)n$,
- (c) $W'(X)$, $W''(X)$, $W'''(X)$ and $W^{iv}(X)$ are continuous.

From the definition, a quintic spline takes the form

$$W(X) = a_0 + \sum_{i=1}^4 a_i (X - X_0)^i + \sum_{j=0}^{n-1} b_j (X - X_j)_+^5 \quad (13)$$

where

$$(X - X_j)_+ = \begin{cases} 0 & \text{if } X < X_j \\ X - X_j & \text{if } X \geq X_j \end{cases} \quad (14)$$

It is also assumed, for simplicity, that the knots X_i are equally spaced in $(0, l)$ with the spacing interval ΔX , so that

$$\Delta X = l/n, \quad X_i = i\Delta X \quad (i=0, 1, 2, \dots, n). \quad (15)$$

The number of unknown constants in equation (13) is $(n + 5)$. Satisfaction of differential equation (11) by collocation at the $(n + 1)$ knots in the interval $(0, l)$ together with the boundary conditions (to be explained in the next section) gives precisely the requisite number of equations for the determination of unknown constants.

Substituting $W(X)$ from equation (13) into equation (11) gives, for satisfaction at the m^{th} knot, one obtains

$$B_4 a_0 + [B_4 (X_q - X_0) + B_3] a_1 + [B_4 (X_q - X_0)^2 + 2B_3 (X_q - X_0) + 2B_2] a_2 + [B_4 (X_q - X_0)^3 + 3B_3 (X_q - X_0)^2 + 6B_2 (X_q - X_0) + 6B_1] a_3 + [B_4 (X_q - X_0)^4 + 4B_3 (X_q - X_0)^3 + 12B_2 (X_q - X_0)^2 + 24B_1 (X_q - X_0) + 24B_0] a_4 + \sum_{i=0}^{n-1} [B_4 (X_q - X_i)^5 + 5B_3 (X_q - X_i)^4 + 20B_2 (X_q - X_i)^3 + 60B_1 (X_q - X_i)^2 + 120B_0 (X_q - X_i)] b_i = 0 \quad (16)$$

where

$$\begin{aligned} B_0 &= [1 - \alpha \cos(\pi/2)X_q] (1 + \beta_1 X_q + \beta_2 X_q^2)^2, \\ B_1 &= 2[\alpha(\pi/2) \sin((\pi/2)X_q) (1 + \beta_1 X_q + \beta_2 X_q^2)^2 + 3(1 - \alpha \cos(\pi/2)X_q) (1 + \beta_1 X_q + \beta_2 X_q^2)(\beta_1 + 2\beta_2 X_q)], \\ B_2 &= [(\alpha(\pi/2)^2 \cos((\pi/2)X_q) (1 + \beta_1 X_q + \beta_2 X_q^2)^2 + 6\alpha(\pi/2) \sin((\pi/2)X_q) (1 + \beta_1 X_q + \beta_2 X_q^2)(\beta_1 + 2\beta_2 X_q) + 6(1 - \alpha \cos(\pi/2)X_q) (\beta_1 + 2\beta_2 X_q)^2 + 6(1 - \alpha \cos(\pi/2)X_q) (\beta_1 + \beta_2 X_q^2) \beta_2] - 2r^2 (1 - \alpha \cos(\pi/2)X_q) (1 + \beta_1 X_q + \beta_2 X_q^2)^2], \\ B_3 &= -2r^2 [\alpha(\pi/2) \sin((\pi/2)X_q) (1 + \beta_1 X_q + \beta_2 X_q^2)^2 + 3(1 - \alpha \cos(\pi/2)X_q) (1 + \beta_1 X_q + \beta_2 X_q^2)(\beta_1 + 2\beta_2 X_q)], \\ B_4 &= r^2 [r^2 (1 - \alpha \cos(\pi/2)X_q) (1 + \beta_1 X_q + \beta_2 X_q^2)^2 - v((\alpha(\pi/2)^2 \cos((\pi/2)X_q) (1 + \beta_1 X_q + \beta_2 X_q^2)^2 + 6\alpha(\pi/2) \sin((\pi/2)X_q) (1 + \beta_1 X_q + \beta_2 X_q^2)(\beta_1 + 2\beta_2 X_q) + 6(1 - \alpha \cos(\pi/2)X_q) (\beta_1 + 2\beta_2 X_q)^2 + 6(1 - \alpha \cos(\pi/2)X_q) (\beta_1 + \beta_2 X_q^2) \beta_2)] - \lambda^2]. \end{aligned}$$

Thus, one obtains a homogeneous set of equations in terms of unknown constants $a_0, a_1, a_2, a_3, a_4, b_0, b_1, \dots, b_{n-1}$, which, when written in matrix notation, takes the form

$$[B][C] = 0 \quad (17)$$

where $[B]$ is an $(n+1) \times (n+5)$ matrix and $[C]$ is an $(n+5) \times 1$ matrix.

4. Boundary Conditions and Frequency Equations

The frequency equations for clamped (C) and simply supported (S) rectangular plates have been obtained by

employing the appropriate boundary conditions:

C-S-C-S- Plates

For a rectangular plate clamped at both the edges $X=0$ and $X=1$ (and simply supported at the remaining two edges),

$$W|_{X=0,1} = (\partial W / \partial X)|_{X=0,1} = 0 \quad (18)$$

Applying the boundary conditions (18), to the deflection function (13), at the two edges $X=0$ and $X=1$, one obtains a set of four homogeneous equations in terms of unknown constants, which can be written as

$$[A_1][C] = 0 \quad (19)$$

where $[A_1]$ is an $4 \times (n+5)$ matrix and $[C]$ is an $(n+5) \times 1$ matrix.

Equation (19), taken together with the equation (17), gives

a complete set of $(n+5)$ equations for a C-S-C-S- plate. These can be written as

$$[B/A_1][C]=0 \quad (20)$$

For a non-trivial solution of equation (20), the characteristic determinant must vanish:

$$|B/A_1|=0 \quad (21)$$

This is the frequency equation for a C-S-C-S- plate.

S-S-S-S- Plates

For a rectangular plate simply supported at both the edges $X=0$ and $X=1$ (and simply supported at the remaining two edges),

$$W|_{X=0,1} = (\partial^2 W / \partial X^2)|_{X=0,1} = 0 \quad (22)$$

Employing the boundary conditions (22), to the deflection function (13), at the two edges $X=0$ and $X=1$, one gets the boundary equations for S-S-S-S- plate as

$$[A_2][C]=0 \quad (23)$$

where $[A_2]$ is an $4 \times (n+5)$ matrix and $[C]$ is an $(n+5) \times 1$ matrix.

Hence frequency equation comes out for S-S-S-S- plate as

$$|B/A_2|=0 \quad (24)$$

5. Results and Discussion

Frequency equations (21) and (24) are transcendental equations in λ^2 from which infinitely many roots can be obtained. The frequency parameter λ corresponding to first three modes of vibration of C-S-C-S- and S-S-S-S- rectangular plates have been computed for $m=1$ and various values of thermal constant (α), taper constants (β_1, β_2) for fixed aspect ratio ($a/b=1.5$). The value of Poisson ratio ν has been taken as 0.3.

To choose the appropriate interpolation interval ΔX , the computer programme has been developed for the evaluation of the frequency parameter λ and run for $n=10(5)60$. The numerical values show a consistent improvement with the increase of the number of knots. In all the above computation,

authors have fixed $n=50$, since further increase in n does not improve the results except in the fifth or sixth decimal places. These results have been tabulated in tables 1 to 3.

The results presented in table 1 show a marked effect of variation of thermal constant (α) on the frequency parameter (λ) for different combinations of taper constant (β_1, β_2) and fixed aspect ratio ($a/b = 1.5$) corresponding the first three modes of vibration for C-S-C-S- and S-S-S-S- plates. The value of frequency parameter decrease with the increase of thermal constant for both the boundary conditions, considered here. Further, it can be seen from table that frequency parameter, for both the boundary conditions, decreases gradually in the third mode of vibrations in comparison to first two modes of vibration.

Table 2 shows the variation of frequency parameter (λ) with taper constant (β_1) for taper constant ($\beta_2=0.5$), two values of thermal constant ($\alpha=0.0, 0.4$) and fixed aspect ratio ($a/b = 1.5$) corresponding to the first three modes of vibration. It is observed that the frequency parameter increases with the increase of taper constant for both the boundary conditions, considered here.

In table 3, the effect of taper constant (β_2) on frequency parameter for taper constant ($\beta_1=0.5$), two values of thermal constant ($\alpha=0.0, 0.4$) and fixed aspect ratio ($a/b = 1.5$) corresponding to the first three modes of vibration for C-S-C-S- and S-S-S-S- plates, have been shown. From this table, one can observe that frequency parameter in first three modes of vibration increases with the increase of taper constant for C-S-C-S- and S-S-S-S- plates.

Further, it can be seen from tables 2 and 3 that frequency parameter, for both the boundary conditions, increases gradually in the third mode of vibrations in comparison to first two modes of vibration. Also, one can observe from tables 1 to 3, that frequency parameter of C-S-C-S- plate is higher than that of S-S-S-S- plate.

Table 1. Value of frequency parameter (λ) for different values of thermal constant (α) with different combinations of taper constant (β_1, β_2) and fixed aspect ratio ($a/b = 1.5$) for C-S-C-S- and S-S-S-S- plate for first three modes of vibrations : $m=1$

β_1, β_2	α	C-S-C-S- PLATE			S-S-S-S- PLATE		
		First Mode	Second Mode	Third Mode	First Mode	Second Mode	Third Mode
$\beta_1 = -0.5, \beta_2 = -0.5$	0.0	29.3011	63.0984	111.0257	21.4941	53.0078	91.1043
	0.1	28.4086	61.1705	107.8041	20.5062	50.8198	88.0017
	0.2	27.5011	58.9103	103.0475	19.4190	48.4502	84.9101
	0.3	26.4327	56.9012	99.5452	18.3208	46.1755	81.8069
	0.4	25.3217	54.8031	95.4472	17.1904	44.0003	78.7043
	0.5	24.2572	52.5301	91.2845	15.9026	41.9805	75.3190
$\beta_1 = -0.5, \beta_2 = 0.5$	0.0	36.0132	72.5490	127.4781	27.3761	63.7524	108.1562
	0.1	35.1201	70.4111	123.1501	26.3401	61.3320	104.7054
	0.2	34.1002	68.3661	119.0776	25.5087	59.1415	101.4532
	0.3	33.0987	66.2145	115.0008	24.4181	57.0097	98.2107
	0.4	31.9908	64.0063	110.8147	23.3011	54.8086	95.0267
	0.5	30.8838	61.9016	106.8064	22.1509	52.5485	91.7213
$\beta_1 = 0.5, \beta_2 = 0.5$	0.0	49.4210	106.8851	191.4330	39.7562	97.2203	171.4612
	0.1	48.5231	104.7087	187.7174	38.6431	95.0083	168.0890
	0.2	47.5109	102.4481	184.7001	37.6304	93.0001	165.0093
	0.3	46.4211	100.1352	181.4697	36.6012	90.8034	161.7973
	0.4	45.2991	97.9103	177.2987	35.5891	88.7282	158.7087
	0.5	44.1267	95.9035	173.0367	34.5327	86.5578	155.4898

Table 2. Value of frequency parameter (λ) for different values of taper constant (β_1) with different combinations of thermal constant (α) and fixed aspect ratio ($a/b = 1.5$) for C-S-C-S- and S-S-S-S- plate for first three modes of vibrations : $m=1$ and $\beta_2=0.5$

α	β_1	C-S-C-S- PLATE			S-S-S-S- PLATE		
		First Mode	Second Mode	Third Mode	First Mode	Second Mode	Third Mode
0.0	-0.5	36.0132	72.5490	127.4781	27.3761	63.7524	108.1562
	-0.3	38.3782	78.1352	137.9481	29.4861	69.2189	118.7010
	-0.1	40.5601	83.7103	148.5275	31.5908	74.7902	129.0211
	0.0	42.5324	89.0914	158.7945	33.3920	79.8053	138.8386
	0.1	44.8210	94.5601	169.4642	35.3904	85.2100	149.0842
	0.3	47.0572	100.4805	180.3443	37.3026	91.0409	160.0490
	0.5	49.4210	106.8851	191.4330	39.7562	97.2203	171.4612
0.4	-0.5	31.9908	64.0063	110.8147	23.3011	54.8086	95.0267
	-0.3	34.1204	69.5261	122.0076	25.4287	60.4120	105.5731
	-0.1	36.3008	75.2896	133.0310	27.4981	65.5974	116.0851
	0.0	38.1209	80.1506	143.6610	29.1311	70.7062	125.8226
	0.1	40.4023	86.1097	156.8074	31.1909	76.0821	136.3921
	0.3	42.8110	91.9081	165.8030	33.2562	82.2520	147.4501
	0.5	45.2891	97.8710	177.4687	35.5891	88.7882	158.8908

Table 3. Value of frequency parameter (λ) for different values of taper constant (β_2) with different combinations of thermal constant (α) and fixed aspect ratio ($a/b = 1.5$) for C-S-C-S- and S-S-S-S- plate for first three modes of vibrations : $m=1$ and $\beta_1=0.5$

α	β_2	C-S-C-S- PLATE			S-S-S-S- PLATE		
		First Mode	Second Mode	Third Mode	First Mode	Second Mode	Third Mode
0.0	-0.5	37.5320	77.7213	142.9941	27.8301	65.7224	116.9956
	-0.3	39.4712	82.2052	151.0998	29.7861	70.8189	126.0010
	-0.1	41.4011	86.9910	159.2275	31.7408	75.9702	135.1221
	0.0	43.1534	91.1293	167.0042	33.4221	80.7053	143.6186
	0.1	45.1210	96.0601	175.1864	35.3090	86.0030	152.7484
	0.3	47.1572	101.2805	183.2443	37.3026	91.3409	162.0060
	0.5	49.4210	106.8851	191.4330	39.7562	97.2203	171.4612
0.4	-0.5	33.2623	75.9919	126.9118	23.6301	60.1092	104.9880
	-0.3	35.1042	79.7146	135.5776	25.6587	64.9321	113.9971
	-0.1	37.1908	83.4846	143.9910	27.7181	69.7807	122.8985
	0.0	38.8088	86.8556	152.1087	29.4611	74.2301	131.2026
	0.1	40.8702	90.5097	160.9904	31.4109	78.9921	140.3021
	0.3	43.1001	94.2951	169.1321	33.4262	83.6220	149.3621
	0.5	45.2991	97.9103	177.2987	35.5891	88.7282	158.7087

6. Conclusions

It can be concluded from the results that frequency parameter increases with increase in taper constants and decreases with increase in thermal gradient. Also, it is evident from the tables 2 and 3 that when $\beta_1=0.5$, the values of frequency parameter more in comparison to $\beta_2=0.5$ for all the three modes of vibrations and both the boundary conditions. It is also clear from the tables that third mode of vibration changes more sharply than second and first.

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