American Journal of Computational and Applied Mathematics 2013, 3(2): 97-105
DOI: 10.5923/j.ajcam.20130302.07

Analytical Soliton Solutions and Wave Solutions of Discrete Nonlinear Cubic-Quintique Ginzburg-Landau Equations in Array of Dissipative Optical Systems

Jean Roger Bogning

Department of Physics, Higher Teacher's Training College, University of Bamenda, PO Box 39, Bamenda, Cameroon

Abstract In this work, we are looking for the wave solutions and soliton solutions of some discrete nonlinear partial differential equations in order to integrate the discrete variable "n" in the expression of amplitude and phase. Hence We choose for this reason the networks of dispersive optical systems where the dynamics of propagation of waves is governed by the discrete nonlinear partial differential equations of Ginzburg-Landau types. With the help of some innovative and suitable transformations, we uncouple the equations to solve in order to get the ones that govern the dynamics of phase and amplitude, hereby facilitating the achievement of our goal. The expressions of the non linear phases are also obtained as well.

Keywords Discrete , Nonlinear, Ginzburg-Landau, Phase Soliton, Pulse Soliton, Soliton Solutions

1. Introduction

Great deals of physical systems are governed by mathematical equations, notably the linear differential equations, linear partial differential equations, and the discrete nonlinear partial differential equations or discrete nonlinear ordinary equations which describe the complicated physical systems. A better knowledge of these physical systems, require the numerical resolution and above all the analytical resolution of these equations. This justifies especially the determination of physicists to continue their research of the physical solutions, or the solutions which are closed to reality. In the last few decades, many authors have been interested in the discrete model equations[1-5]. The best techniques of resolution have been discovered or improved; others have been used to give the solutions of different mathematical and nonlinear physical equations [6-18]. Very often it is the future wave named soliton that is in the center of several nonlinear partial differential equations resolution.

It has caused a lot of interest in the minds of most researchers resulting in a series of publications[19-25]. In optical fibers and other domains, researchers brought forward the solutions of the equations and treat the nonlinear dynamic excitations[26-38]. In short the interest of the researchers in resolving of the partial differential equations and in the nonlinear partial differential equations is more and more crescent according. If the spatio-temporal nonlinear partial differential equations (NPDE) is not comfortable to solve, the one of the discrete NPDE is even more complicated. It is due to the fact that in addition to the spatial and temporal variables, we must use another variable which determines the number of elements that the network contains. Speaking of the network, a great deal of the discrete NPDEs describe the dynamics of propagation of waves in the networks like optical fibers, atomic chains, and electric lines,… and so on and so forth. But the problem remains how to find wave solutions of the shape $\psi_n = U(M,t,n)\exp i\theta_n(M,t,n)$ where $U(M,t,n)$ represents the envelope of the wave and $\theta_n(M,t,n)$ the phase, $M(x,y,z)$ a point of space, n the discrete variable and t the time. i.e. to find a solution that takes into account the discrete variable in the amplitude. If a lot of research works yields many solutions under the shape $U(M,t)\exp i\theta_n(M,t,n)$ where the amplitude is a function of space and time, there are few cases where the amplitude is a function of space, time and discrete variable n.

In our previous works, we demonstrated that it was possible to transform the discrete NPDE with the help of the recurrence relations which govern the phase[37]. This method will be put in evidence in this work that is to look for the solutions of the discrete complex Ginzburg-Landau equations in dissipative mode. The NPDEs considered here take into account the temporal and discrete variable. The

* Corresponding author:
rbogning@yahoo.com (Jean Roger Bogning)
Published online at http://journal.sapub.org/ajcam

solutions that we are going to propose will have the shape $\psi_n = U(n,t)\exp i\theta_n(n,t)$ where n is the discrete variable and t the time.

Hereafter, we find the analytical soliton solutions of the discrete cubic-quintique nonlinear and complex Ginzburg-Landau equation by the right method (see ref.[37]). This method consists of bounding the equation by the lower bound and upper bound equations, and after obtaining the induction relation which governs the phases of the solutions. We obtained these induction relations by finding the values which give out $\cos\left(\frac{\theta_{n+1}-\theta_{n-1}}{2}\right)$, $\cos\left(\frac{\theta_{n+1}+\theta_{n-1}-2\theta_n}{2}\right)$ and $\sin\left(\frac{\theta_{n+1}+\theta_{n-1}-2\theta_n}{2}\right)$ max. Then the solutions obtained take into account the discrete number "n", the real time of propagation "t" and the time delay due to the influence of other fibers on the considered propagating fiber.

This paper is organized as follows. In section 2, we introduced the method to be used, in section 3, we apply this method on Ablowitz-Ladik's equation which is a single nonlinear cubic and integrable Schrödinger equation. Section 4, proposes the analytical solutions of the dissipative discrete cubic nonlinear complex Ginzburg-Landau equation. Section 5, proposes the analytical solutions of the dissipative discrete cubic-quintique nonlinear complex Ginzburg-Land au[25], section 6 proposes the analytical solutions of the discrete cubic nonlinear complex Ginzburg-Landau equation in the dissipative model with nonlinear terms $|\psi_n|^2(\psi_{n+1}+\psi_{n-1})$ and $|\psi_n|^4(\psi_{n+1}+\psi_{n-1})$. Finally, section 7 concludes with brief comments.

2. Method

The main principle behind this method consists of introducing in the equation to solve as a solution of the shape $\psi_n = U(n,t)\exp i\theta_n(n,t)$

$\psi_n = U(n,t)\exp i\theta_n(n,t)$. It leads to coupled equations in θ_n and U. While transforming $\exp i\theta_n$ according to the Moivre's formula, the resulting equations can be shown under shapes that include the terms in $\cos\left(\frac{\theta_{n+1}+\theta_{n-1}-2\theta_n}{2}\right)\cos\left(\frac{\theta_{n+1}-\theta_{n-1}}{2}\right)$, $\sin\left(\frac{\theta_{n+1}+\theta_{n-1}-2\theta_n}{2}\right)\cos\left(\frac{\theta_{n+1}-\theta_{n-1}}{2}\right)$ or $\cos\left(\frac{\theta_{n+1}+\theta_{n-1}-2\theta_n}{2}\right)\sin\left(\frac{\theta_{n+1}-\theta_{n-1}}{2}\right)$,..., and so on and so forth. The presence of the expressions of this nature, allows obtaining the bounded equations which are limited by two other equations of which one is lower bound and the other is upper bound. Following these observations, we note that to have the lower bound equation or upper bound equation result in the following correspondences $\left|\cos\left(\frac{\theta_{n+1}+\theta_{n-1}-2\theta_n}{2}\right)\cos\left(\frac{\theta_{n+1}-\theta_{n-1}}{2}\right)\right| \to 1$, $\left|\cos\left(\frac{\theta_{n+1}+\theta_{n-1}-2\theta_n}{2}\right)\cos\left(\frac{\theta_{n+1}-\theta_{n-1}}{2}\right)\right| \to 0$ or $\left|\sin\left(\frac{\theta_{n+1}+\theta_{n-1}-2\theta_n}{2}\right)\cos\left(\frac{\theta_{n+1}-\theta_{n-1}}{2}\right)\right| \to 1$ and so forth. These correspondences permit the recurrence relations that govern θ_n of which the resulting equations, end with the expressions of $U(n,t)$ and θ_1 which are the first term of θ_n when $n=1$ ($\theta_1=\theta_1(t)$). What is also important to mention here is that, we want amplitude $U(n,t)$ that is effectively a function of n. To arrive there, we put the change of variable $T_n = t+(n-1)\tau$ where t is the effective time cover by the wave in a considered propagator fiber or cell of the network without the influence of the neighbors, τ the time delay imposed by the neighboring fibers or neighboring cells on the propagator fiber or cell if one supposes that the contribution of every fiber is the same and is worth τ, n represents the number of fibers or cells that the network contains. T_n can take various shapes, for example $T_n = t+2\left[1-(1/2)^n\right]\tau$ if the delay time imposed by every fiber or cell varies according to the remoteness with respect to the propagator fiber as a geometric regression of common ratio $q=1/2$.

In the course of this work, we are going to use this method to propose solutions of some discrete NPDEs of Ginzburg-Landau in dissipative media.

3. Cubic Nonlinear Schrödinger Equation

We use the cubic integrable Schrödinger equation (Ablowitz-Ladik)[38] given by the following relation

$$i\frac{d\psi_n}{dt}+\frac{D}{2}(\psi_{n+1}+\psi_{n-1}-2\psi_n) + |\psi_n|^2(\psi_{n+1}+\psi_{n-1})=0 \quad (1)$$

Using the transformation $T_n = t+(n-1)\tau$, where n is the number of fibers or cores, τ the time delay due to one fiber or core and t the real time of propagation in the absence of neighboring fibers. The equation (1) becomes

$$i\frac{d\psi_n}{dT_n}+\frac{D}{2}(\psi_{n+1}+\psi_{n-1}-2\psi_n)+|\psi_n|^2(\psi_{n+1}+\psi_{n-1})=0. \quad (2)$$

Looking for the solutions of equation (2) in the form

$$\psi_n = U(n,t)\exp(i\theta_n) = U(T_n)\exp(i\theta_n) \quad (3)$$

we reduce equation (2) as

$$\begin{aligned}&-U\frac{d\theta_n}{dT_n}+\frac{D}{2}\left[\cos(\theta_{n+1}-\theta_n)+\cos(\theta_{n-1}-\theta_n)-2\right]U\\&+\left[\cos(\theta_{n+1}-\theta_n)+\cos(\theta_{n-1}-\theta_n)\right]|U|^2U\\&+i\left\{\begin{aligned}&\frac{dU}{dT_n}+\frac{D}{2}\left[\sin(\theta_{n+1}-\theta_n)+\sin(\theta_{n-1}-\theta_n)\right]U\\&+\left[\sin(\theta_{n+1}-\theta_n)+\sin(\theta_{n-1}-\theta_n)\right]|U|^2U\end{aligned}\right\}=0,\end{aligned} \quad (4)$$

then we end up with the dynamic coupled equations by decomposing into real and imaginary parts the equation (4)

$$\begin{aligned}&-U\frac{d\theta_n}{dT_n}+DU\cos\left(\frac{\theta_{n+1}+\theta_{n-1}-2\theta_n}{2}\right)\cos\left(\frac{\theta_{n+1}-\theta_{n-1}}{2}\right)\\&-DU+2|U|^2U\cos\left(\frac{\theta_{n+1}+\theta_{n-1}-2\theta_n}{2}\right)\cos\left(\frac{\theta_{n+1}-\theta_{n-1}}{2}\right)=0,\end{aligned} \quad (5)$$

and

$$\begin{aligned}&\frac{dU}{dT_n}+DU\cos\left(\frac{\theta_{n+1}-\theta_{n-1}}{2}\right)\sin\left(\frac{\theta_{n+1}+\theta_{n-1}-2\theta_n}{2}\right)\\&+2|U|^2U\cos\left(\frac{\theta_{n+1}-\theta_{n-1}}{2}\right)\sin\left(\frac{\theta_{n+1}+\theta_{n-1}-2\theta_n}{2}\right)=0.\end{aligned} \quad (6)$$

While proceeding by a framing of the equation (5) and equation (6), we observed that

$$equation(5) \le -U\frac{d\theta_n}{dT_n}+2|U|^2U, \quad (7)$$

and

$$\frac{dU}{dT_n}-DU-2|U|^2U \le equation(6) \le \frac{dU}{dT_n}+DU+2|U|^2U. \quad (8)$$

In other words, the limitation of these equations becomes to obtain their upper bound and lower bound. We are tempted to say that the above framings are obtained when terms in cosine and sine of θ_n are maximum or minimum. If we consider the case where $\left|\cos\left(\frac{\theta_{n+1}+\theta_{n-1}-2\theta_n}{2}\right)\cos\left(\frac{\theta_{n+1}-\theta_{n-1}}{2}\right)\right| \to 1$, then the following relations are verified

$$\theta_{n+1}+\theta_{n-1}-2\theta_n = 2n\pi, \quad (9)$$

and

$$\theta_{n+1}-\theta_n = 2n\pi. \quad (10)$$

The addition of equation (9) and equation (10) leads to recurrence relation

$$\theta_{n+1}-\theta_n = 2n\pi. \quad (11)$$

When n varies the equation (11) yields

$$\theta_n = \theta_1 + \pi(n^2-n), \quad (12)$$

where θ_1 represents the expression of θ_n when $n=1$.

The substitution of equation (12) into equation (5) and equation (6) yields the reduced system of equations

$$\frac{d\theta_1}{dT_n} = 2U^2, \quad (13)$$

$$\frac{dU}{dT_n} = 0. \quad (14)$$

The integration of equation (14) and equation (13) gives $U = U_0 = cste$, $\theta_1 = 2U_0^2 t + \eta$ (because for $n=1, T_n \to t$). In this case the solution of equation (1) is given by

$$\psi_n = U_0 \exp i\left(2U_0^2 T_n + \pi(n^2-n)+\eta\right). \quad (15)$$

In the second case, $\left|\cos\left(\frac{\theta_{n+1}-\theta_{n-1}}{2}\right)\sin\left(\frac{\theta_{n+1}+\theta_{n-1}-2\theta_n}{2}\right)\right| \to 1$, then we obtain

$$\theta_{n+1}+\theta_{n-1}-2\theta_n = (2n+1)\pi, \quad (16)$$

and

$$\theta_{n+1}-\theta_n = 2n\pi. \quad (17)$$

With the same logic adding equation (16) to equation (17), we obtain

$$\theta_n = \theta_1 + \frac{\pi}{2}\sum_{k=1}^{n-1}(4k-3), \quad (18)$$

Where n describes the set of natural numbers and θ_1 is the expression of θ_n when $n=1$. When n varies equation (18) is written

$$\theta_n = \theta_1 + \frac{\pi}{2}\sum_{k=1}^{n-1}(4k-3). \quad (19)$$

Taking into account equation (19) in equation (6), it follows that

$$\frac{d\theta_1}{dT_n} = -D, \quad (20)$$

and

$$\frac{dU}{dT_n} + DU + 2U^3 = 0. \quad (21)$$

The integration of equation (20) gives $\theta_1 = -DT_n + \lambda_1$, where λ_1 is an arbitrary constant, $T_n = t$ because $n=1$ and the equation (19) becomes

$$\theta_n = -Dt + \frac{\pi}{2}\sum_{k=1}^{n}(4k-3) + \lambda_1. \quad (22)$$

It follows from the resolution of equation (21) that

$$U(T_n) = \frac{\pm\beta\sqrt{D}\,\sec h(DT_n)}{\sqrt{2+\tanh(DT_n)-2\sec h^2(DT_n)}}, \quad (23)$$

where β is an arbitrary constant and $D \succ 0$. From equations (1), (2), (22) and equation (23) we can therefore write the solution of equation (1) as

$$U(T_n)=\frac{\pm\beta\sqrt{D}\sec h\left(D(t-n\tau)\right)}{\sqrt{2+\tanh(D(t-n\tau))-2\sec h^2\left(D(t-n\tau)\right)}}\times\exp i\left(-Dt+\frac{\pi}{2}\sum_{k=1}^{n}(4k-3)+\lambda_1\right). \quad (24)$$

The used of the above method permits to really uncouple the equation (1); at the end we obtain the solution of equation (24).

As we have shown previously, it is possible to get several relations of recurrences that manage the phase θ_n. But in the course of this work, we are going to consider that θ_n is under the shape $\theta_n=\theta_1+\pi\left(n^2-n\right)$, where $\theta_1=\theta_1(t)$.

4. Solutions of Dissipative Cubic Nonlinear Complex Ginzburg-Landau Equation

In this section, we consider the cubic nonlinear complex Ginzburg-landau equation in the dissipative system[24] given by

$$i\frac{d\psi_n}{dt}+\left(\frac{D}{2}-i\beta\right)\left(\psi_{n+1}+\psi_{n-1}-2\psi_n\right)+(1-i\varepsilon)\left|\psi_n\right|^2\left(\psi_{n+1}+\psi_{n-1}\right)=i\delta\psi_n. \quad (25)$$

Using the transformation $T_n=t+(n-1)\tau$, we rewrite equation (25) in the form

$$i\frac{d\psi_n}{dT_n}+\left(\frac{D}{2}-i\beta\right)\left(\psi_{n+1}+\psi_{n-1}-2\psi_n\right)+(1-i\varepsilon)\left|\psi_n\right|^2\left(\psi_{n+1}+\psi_{n-1}\right)=i\delta\psi_n. \quad (26)$$

Inserting equation (3) in equation (26) and applying the same transformation we obtain

$$-U\frac{d\theta_n}{dT_n}-DU+\left(DU+2|U|^2U\right)\times\cos\left(\frac{\theta_{n+1}+\theta_{n-1}-2\theta_n}{2}\right)\cos\left(\frac{\theta_{n+1}-\theta_{n-1}}{2}\right)+2\left(\beta U+\varepsilon|U|^2U\right)\times\cos\left(\frac{\theta_{n+1}-\theta_{n-1}}{2}\right)\sin\left(\frac{\theta_{n+1}+\theta_{n-1}-2\theta_n}{2}\right)=0, \quad (27)$$

and

$$\frac{dU}{dT_n}+(2\beta-\delta)U+\left(DU+2|U|^2U\right)\times\cos\left(\frac{\theta_{n+1}+\theta_{n-1}-2\theta_n}{2}\right)\cos\left(\frac{\theta_{n+1}-\theta_{n-1}}{2}\right)+2\left(\beta U+\varepsilon|U|^2U\right)\times\cos\left(\frac{\theta_{n+1}-\theta_{n-1}}{2}\right)\sin\left(\frac{\theta_{n+1}+\theta_{n-1}-2\theta_n}{2}\right)=0. \quad (28)$$

The substitution of equation (12) into equation (27) and equation (28) reduced them to

$$\frac{d\theta_1}{dT_n}+2U^2=0, \quad (29)$$

and

$$\frac{dU}{dT_n}+(2\beta-\delta+D)U+2U^3=0. \quad (30)$$

Set down $\Omega_0=2\beta-\delta+D$, and according to the fact that $U=U(T_n)$ we rewrite equation (29) and equation (30) as

$$\frac{d\theta_1}{dT_n}+2U^2=0, \quad (31)$$

and

$$\frac{dU}{dT_n}+\Omega_0U+2U^3=0. \quad (32)$$

The integration of equation (32) leads to

$$U(T_n)=\frac{\pm\sqrt{\Omega_0}\sec h(\Omega_0T_n)}{\sqrt{2C_2\Omega_0\left(1+\tanh(\Omega_0T_n)\right)-(2+\Omega_0C_2)\sec h^2(\Omega_0T_n)}}. \quad (33)$$

By introducing equation (33) into equation (31) we obtain the expression of the phase for $n=1$

$$\theta_0=\frac{1}{2}\ln\frac{2+2\tanh(\Omega_0t)-\sec h^2(\Omega_0t)}{\sqrt{2C_2\Omega_0\left(1+\tanh(\Omega_0t)\right)-(2+\Omega_0C_2)\sec h^2(\Omega_0t)}}+\lambda_2, \quad (34)$$

where $T_n=t$, C_2, λ_2 are the arbitrary constants. From equation (34) and equation (12) we obtain the following relation

$$\theta_n=\frac{1}{2}\ln\frac{2+2\tanh(\Omega_0t)-\sec h^2(\Omega_0t)}{\sqrt{2C_2\Omega_0\left(1+\tanh(\Omega_0t)\right)-(2+\Omega_0C_2)\sec h^2(\Omega_0t)}}+\pi\left(n^2-n\right)+\lambda_2. \quad (35)$$

According to equation (35) and equation (33), the solution of equation (25) can be written in the following form

$$\psi_n(t,n)=\left(\frac{\pm\sqrt{\Omega_0}\sec h\left(\Omega_0\left(t-n\tau\right)\right)}{\sqrt{\begin{array}{l}2C_2\Omega_0\left(1+\tanh\left(\Omega_0\left(t-n\tau\right)\right)\right)\\-\left(2+\Omega_0C_2\right)\sec h^2\left(\Omega_0\left(t-n\tau\right)\right)\end{array}}}\right) \quad (36)$$
$$\times\exp\left(\begin{array}{l}\frac{1}{2}\ln\frac{2+2\tanh\left(\Omega_0t\right)-\sec h^2\left(\Omega_0t\right)}{\sqrt{\begin{array}{l}2C_2\Omega_0\left(1+\tanh\left(\Omega_0t\right)\right)\\-\left(2+\Omega_0C_2\right)\sec h^2\left(\Omega_0t\right)\end{array}}}\\+\pi\left(n^2-n\right)+\lambda_2\end{array}\right).$$

In the next section, we are going to use the same method and transformation to seek the analytical solutions of the complex Ginzburg-Landau equation with high nonlinearity.

5. Solutions of Dissipative Complex Cubic-Quintique Nonlinear Ginzburg-Landau Equation

The equation that we consider here is practically the one that we used in section 3 but with quintique nonlinearity. This equation is given by

$$\begin{array}{l}i\frac{d\psi_n}{dt}+\left(\frac{D}{2}-i\beta\right)\left(\psi_{n+1}+\psi_{n-1}-2\psi_n\right)\\+2\left(1-i\varepsilon\right)\left|\psi_n\right|^2\psi_n\\+2\left(\gamma-i\mu\right)\left|\psi_n\right|^4\psi_n=i\delta\psi_n.\end{array} \quad (37)$$

Thus, using the transformation $T_n=t-\left(n-1\right)\tau$, it follows from equation (37) that

$$\begin{array}{l}i\frac{d\psi_n}{dT_n}+\left(\frac{D}{2}-i\beta\right)\left(\psi_{n+1}+\psi_{n-1}-2\psi_n\right)\\+2\left(1-i\varepsilon\right)\left|\psi_n\right|^2\psi_n\\+2\left(\gamma-i\mu\right)\left|\psi_n\right|^4\psi_n=i\delta\psi_n.\end{array} \quad (38)$$

Equation (38) leads accordingly to equation (3) the set of coupled equations

$$\begin{array}{l}\frac{d\theta_n}{dT_n}-2\left|U\right|^2-2\gamma\left|U\right|^4+D\\-D\cos\left(\frac{\theta_{n+1}+\theta_{n-1}-2\theta_n}{2}\right)\cos\left(\frac{\theta_{n+1}-\theta_{n-1}}{2}\right)\\-2\beta\cos\left(\frac{\theta_{n+1}+\theta_{n-1}-2\theta_n}{2}\right)\cos\left(\frac{\theta_{n+1}-\theta_{n-1}}{2}\right)=0,\end{array} \quad (39)$$

and

$$\begin{array}{l}\frac{dU}{dT_n}-2\varepsilon\left|U\right|^2-2\left|U\right|^4U-\delta U+2\beta U\\+DU\sin\left(\frac{\theta_{n+1}+\theta_{n-1}-2\theta_n}{2}\right)\cos\left(\frac{\theta_{n+1}-\theta_{n-1}}{2}\right)\\-2\beta U\cos\left(\frac{\theta_{n+1}+\theta_{n-1}-2\theta_n}{2}\right)\cos\left(\frac{\theta_{n+1}-\theta_{n-1}}{2}\right)=0.\end{array} \quad (40)$$

We similarly obtain by substituting equation (12) into equation (39) and (40) the reduced set of the following coupled equations

$$\frac{d\theta_n}{dT_n}-2\gamma\left|U\right|^4-2\left|U\right|^2-2\beta=0, \quad (41)$$

and

$$\frac{dU}{dT_n}-2U^5-2\varepsilon U^3+\Omega_1U=0, \quad (42)$$

where $\Omega_1=-2\beta+\delta$. Looking for general solution of equation (42), it follows that U verify the relation

$$\begin{array}{l}\frac{LnU}{\Omega_1}-\frac{1}{4\Omega_1}\ln\left(2U^4+2\varepsilon U^2+\Omega_1\right)\\+\frac{\varepsilon}{\Omega_1\sqrt{-2\Omega_1+\varepsilon^2}}\arctan\left(\frac{2\varepsilon^2+U^2}{2\sqrt{-2\Omega_1+\varepsilon^2}}\right)=T_n+\lambda_3,\end{array} \quad (43)$$

where λ_3 is the arbitrary constant. To obtain θ_1 in this case, we introduce the value of U given by equation (43) in equation (41). But it will be very difficult to obtain the general expression of θ_1. Nevertheless, we can obtain solutions of equation (41) and (42) by imputing values to different constants ε, Ω_1, γ and β. By considering the case where $\varepsilon=0$, $\gamma\neq0$ and $\Omega_1\neq0$, the system form by equation (41) and (42) becomes

$$\frac{d\theta_1}{dT_n}-2\gamma U^4-2U^2-2\beta=0, \quad (44)$$

and

$$\frac{dU}{dT_n}-2U^5-\Omega_1U=0. \quad (45)$$

Equation (45) admits for solutions

$$U_1=\frac{\Omega_1^{1/4}\sec h^{3/2}\left(2\Omega_1T_n\right)}{\left[\begin{array}{l}2\Omega_1C_3\left(1-\tanh\left(2\Omega_1T_n\right)\right)\\-\left(\Omega_1C_3+2\right)\sec h^2\left(2\Omega_1T_n\right)\end{array}\right]^{1/4}}, \quad (46)$$

$$U_2=\frac{-\Omega_1^{1/4}\sec h^{3/2}\left(2\Omega_1T_n\right)}{\left[\begin{array}{l}2\Omega_1C_3\left(1-\tanh\left(2\Omega_1T_n\right)\right)\\-\left(\Omega_1C_3+2\right)\sec h^2\left(2\Omega_1T_n\right)\end{array}\right]^{1/4}}, \quad (47)$$

$$U_3 = \frac{-\Omega_1^{1/4}\sec h^{3/2}\left(2\Omega_1 T_n\right)}{\left[2\Omega_1 C_3\left(-1+\tanh\left(2\Omega_1 T_n\right)\right)+\left(\Omega_1 C_3+2\right)\sec h^2\left(2\Omega_1 T_n\right)\right]^{1/4}}, \tag{48}$$

$$U_4 = \frac{\Omega_1^{1/4}\sec h^{3/2}\left(2\Omega_1 T_n\right)}{\left[2\Omega_1 C_3\left(-1+\tanh\left(2\Omega_1 T_n\right)\right)+\left(\Omega_1 C_3+2\right)\sec h^2\left(2\Omega_1 T_n\right)\right]^{1/4}}. \tag{49}$$

For every $U_i\left(i=1,...,4\right)$ correspond to $\theta_{1i}\left(i=1,...,4\right)$ and naturally the final expression of ψ_n. Then, taking into account equation (46) into equation (44), we obtain

$$\begin{aligned}\theta_{01} = &\frac{-1}{2}Arc\tan\left[\cosh\left(2\Omega_1 T_n\right)\left(\frac{4C_3\Omega_1^2\left(1-\tanh\left(2\Omega_1 T_n\right)\right)-\left(4\Omega_1+2C_3\Omega_1^2\right)\sec h^2\left(2\Omega_1 T_n\right)}{\Omega_1^2}\right)^{1/2}\right]\\ &+\gamma\ln\left(\frac{\left(2-2\tanh\left(2\Omega_1 T_n\right)\right)^{1/4}\sec h^{1/2}\left(2\Omega_1 T_n\right)}{2\Omega_1 C_3\left(-1+\tanh\left(2\Omega_1 T_n\right)\right)-2\sec h^2\left(2\Omega_1 T_n\right)}\right)-2\beta T_n+\lambda_4,\end{aligned} \tag{50}$$

where C_3, λ_4 are constants, $T_n = t$. From equation (46) and equation (50), we deduce one particular solution of equation (37)

$$\begin{aligned}\psi_n = &\frac{\Omega_1^{1/4}\sec h^{3/2}\left(2\Omega_1\left(t-n\tau\right)\right)}{\left[2\Omega_1 C_3\left(1-\tanh\left(2\Omega_1\left(t-n\tau\right)\right)\right)-\left(\Omega_1 C_3+2\right)\sec h^2\left(2\Omega_1\left(t-n\tau\right)\right)\right]^{1/4}}\\ &\times\exp i\left\{\begin{aligned}&\frac{-1}{2}Arc\tan\left[\left(\frac{4C_3\Omega_1^2\left(1-\tanh\left(2\Omega_1\left(t-n\tau\right)\right)\right)-\left(4\Omega_1+2C_3\Omega_1^2\right)\sec h^2\left(2\Omega_1\left(t\right)\right)}{\Omega_1^2}\right)^{1/2}\cosh\left(2\Omega_1\left(t\right)\right)\right]\\ &+\gamma\ln\left(\frac{\left(2-2\tanh\left(2\Omega_1\left(t\right)\right)\right)^{1/4}\sec h^{1/2}\left(2\Omega_1\left(t\right)\right)}{2\Omega_1 C_3\left(-1+\tanh\left(2\Omega_1\left(t\right)\right)\right)-2\sec h^2\left(2\Omega_1\left(t\right)\right)}\right)-2\beta t+\pi\left(n^2-n\right)+\lambda_4\end{aligned}\right\}.\end{aligned} \tag{51}$$

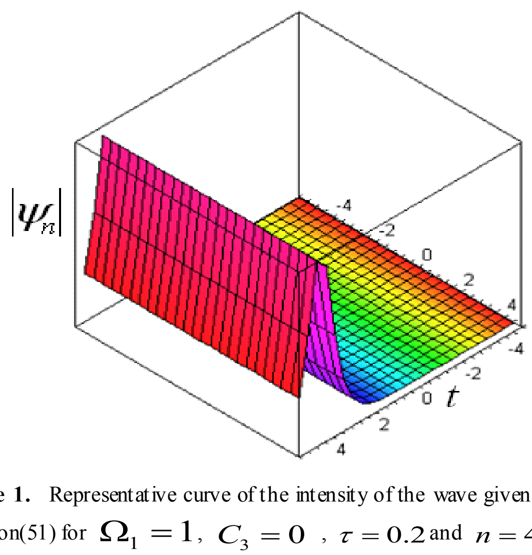

Figure 1. Representative curve of the intensity of the wave given by the equation(51) for $\Omega_1 = 1$, $C_3 = 0$, $\tau = 0.2$ and $n = 41$

Figure 1, shows the profile of intensity of temporal and discrete soliton obtained from the equation (51). For a number of fibers of the network estimated to 41, the choice of the other parameters reduced the analytical expression of the equation (51) to $\psi_{41} = \left(2\right)^{\frac{-1}{4}}\sec h\left(2t+8\right)$; which is logically the analytical shape of a pulse. But when one observes Figure 1, one surrenders to the evidence that it is a pulse that has undergone distortions with respect to a classic pulse. This distortion can be interpreted as due to the influence of the number of fibers " n " that we took into account in the expression of the amplitude of the equation (51).

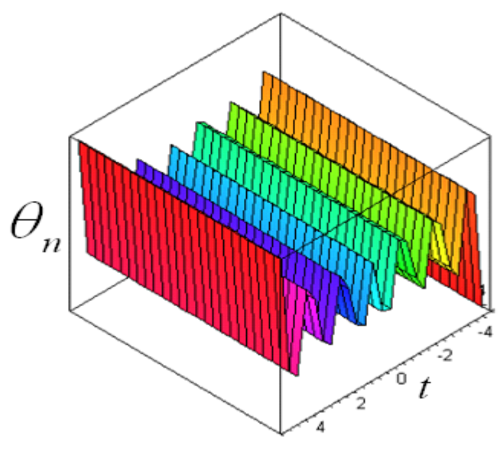

Figure 2. Representative curve of the phase θ_n of the wave given by the equation(51) for $\gamma = 0$, $\beta = 1$, $\lambda_4 = 0$ and $n = 41$

Figure 2, shows the representation of the phase θ_n of the solution given by the equation (51) according to time. We note that this profile of phase is all specific and at such we can ask ourselves the question if we are always right every time and on any type of equation to suppose that the phases of the solutions to construct are linear in temporal variable and in spatial variable?

As we progress with our analysis, we are going to seek solutions of complex cubic-quintique nonlinear Ginzburg-Landau equation which contains the additive terms $|\psi_n|^2(\psi_{n+1}+\psi_{n-1})$ and $|\psi_n|^4(\psi_{n+1}+\psi_{n-1})$.

6. Solutions of Dissipative Complex Cubic-Quintique Nonlinear Ginzburg-Landau Equation Containing the Terms $|\psi_n|^2(\psi_{n+1}+\psi_{n-1})$ and $|\psi_n|^4(\psi_{n+1}+\psi_{n-1})$

In this section of work, the equation which will be in the center of our analysis has the following shape

$$i\frac{d\psi_n}{dt}+\left(\frac{D}{2}-i\beta\right)(\psi_{n+1}+\psi_{n-1}-2\psi_n)+(1+i\varepsilon)|\psi_n|^2(\psi_{n+1}+\psi_{n-1})+(\gamma-i\mu)|\psi_n|^4(\psi_{n+1}+\psi_{n-1})=i\delta\psi_n. \quad (52)$$

Using the same transformations as in the preceding equations, equation (52) gives

$$\begin{aligned}&-U\frac{d\theta_n}{dt}-DU+2\left(\frac{D}{2}U+|U|^2U+\gamma|U|^4U\right)\times\cos\left(\frac{\theta_{n+1}+\theta_{n-1}-2\theta_n}{2}\right)\cos\left(\frac{\theta_{n+1}-\theta_{n-1}}{2}\right)\\&+2\left(\beta U+\varepsilon|U|^2U+\mu|U|^4U\right)\times\cos\left(\frac{\theta_{n+1}-\theta_{n-1}}{2}\right)\sin\left(\frac{\theta_{n+1}+\theta_{n-1}-2\theta_n}{2}\right)=0,\end{aligned} \quad (53)$$

and

$$\begin{aligned}&\frac{dU}{dt}+2\beta U-\delta U+2\left(-\beta U-\varepsilon|U|^2U-\mu|U|^2U\right)\times\cos\left(\frac{\theta_{n+1}+\theta_{n-1}-2\theta_n}{2}\right)\cos\left(\frac{\theta_{n+1}-\theta_{n-1}}{2}\right)\\&+2\left(\frac{D}{2}U+|U|^2U+\gamma|U|^4U\right)\times\cos\left(\frac{\theta_{n+1}-\theta_{n-1}}{2}\right)\sin\left(\frac{\theta_{n+1}+\theta_{n-1}-2\theta_n}{2}\right)=0.\end{aligned} \quad (54)$$

The above equations can be solve by locating and using the expressions of θ_n established at the beginning of this work. So when θ_n takes $\theta_n=\theta_1+\pi/2\sum_{k=1}^{n-1}(4k-3)$, equation (53) and equation (54) yield the system of coupled equations analogous to the system obtained in equation (41) and equation (42) which is already solved . Then we focus our attention in the case where $\theta_n=\theta_1+(n^2-n)\pi$. With this transformation equation (53) and (54) give respectively

$$\frac{d\theta_1}{dT_n}-2\Omega_2U^4-2\alpha U^2+2\beta=0, \quad (55)$$

and

$$\frac{dU}{dT_n}-\delta U-2\Delta U^3=0, \quad (56)$$

where $\alpha=1+\varepsilon$, $\Omega_2=\gamma+\mu$ and $\Delta=\varepsilon+\mu$, we obtain via the resolution of equation (56)

$$U(T_n)=\frac{\pm\sqrt{\delta}\sec h(2\delta T_n)}{\left[2\delta C_4(1-\tanh(\delta T_n))-2\Delta\sec h^2(\delta T_n)\right]^{1/2}}, \quad (57)$$

where C_4 is the arbitrary constant. The substitution of equation (57) into equation (55) leads to

$$\begin{aligned}\theta_0=&\frac{1}{2\Delta}\ln\left(\frac{2(1-\tanh(\delta t))-\sec h^2(\delta t)}{2\delta C_4(1-\tanh(\delta t))-(2\Delta+1)\sec h^2(\delta t)}\right)+\frac{1}{4\Delta^2}\ln\left(\frac{2\delta C_4(1-\tanh(\delta t))-(2\Delta+1)\sec h^2(\delta t)}{2(1-\tanh(\delta t))-\sec h^2(\delta t)}\right)\\&+\frac{\Omega_2\delta\sec h^{-2}(\delta t)}{2\Delta\left[2\delta C_4(1-\tanh(\delta t))-(2\Delta+1)\sec h^2(\delta t)\right]}-2\beta t+\lambda_5,\end{aligned} \quad (58)$$

where λ_5 is an arbitrary constant. Finally the solution of equation (52) is given by

$$\psi_n(t,n) = \frac{\pm\sqrt{\delta}\,\sec h\left(2\delta\left(t-n\tau\right)\right)}{\left[2\delta C_4\left(1-\tanh\left(\delta\left(t\sqrt{n\tau}\right)\right)\right)-2\Delta\sec h^2\left(\delta\left(t-n\tau\right)\right)\right]^{1/2}}$$

$$\times\exp i\left[\begin{array}{l}\dfrac{1}{2\Delta}\ln\left(\dfrac{2\left(1-\tanh\left(\delta t\right)\right)-\sec h^2\left(\delta t\right)}{2\delta C_4\left(1-\tanh\left(\delta t\right)\right)-\left(2\Delta+1\right)\sec h^2\left(\delta t\right)}\right)\\ +\dfrac{1}{4\Delta^2}\ln\left(\dfrac{2\delta C_4\left(1-\tanh\left(\delta t\right)\right)-\left(2\Delta+1\right)\sec h^2\left(\delta t\right)}{2\left(1-\tanh\left(\delta t\right)\right)-\sec h^2\left(\delta t\right)}\right)\\ +\dfrac{\Omega_2\delta\sec h^{-2}\left(\delta t\right)}{2\Delta\left[2\delta C_4\left(1-\tanh\left(\delta t\right)\right)-\left(2\Delta+1\right)\sec h^2\left(\delta t\right)\right]}-2\beta t+\pi\left(n^2-n\right)+\lambda_5,\end{array}\right]. \quad (59)$$

7. Conclusions

The main objective of this research work was to bring forward wave solutions of some discrete nonlinear partial differential equations so that the discrete variable "n" is taken simultaneously into account in the expression of amplitude and phase; what is often very difficult to achieve. We believe that this objective has been attained with different supports of analyses considered hereafter. We want to speak here of Ginzburg-Landau equations presenting the different shapes of nonlinearities (cubic and quintique). Another merit of this work is to enable us to have the phases that are different from the classical phases and linear phases used fluently. The linear phase use fluently is for the shape $\omega t + kz$ or $\omega t + kz + \xi n$. These results indicate merely that it is possible to consider some wave solutions with phases that have other shapes like $\sec h$, $\cos ech$, $\arctan$, $\tanh$, $\ln(\tanh)$,…, and so forth according to the goal that one wishes to attain. Another important report made is that the taking into account of the discrete variable in the amplitude expressions influences its profile considerably. We polish up our work by pointing out that solutions of discrete non linear partial differential equations obtained as in this article can to a greater extend provide with factors that influence the propagation of signals in the transmission network, notably the phenomena of multiple instabilities and intrinsic dissipations.

ACKNOWLEDGEMENTS

We acknowledge support from the ministry of Higher Education of Cameroon through its program of support to research, which enabled us to carry out this work. We also thank Mr Kenfack Christopher Nguimatio for some corrections after the reading of the work.

REFERENCES

[1] D. N. Christodoulides, R.I. Joseph, Discrete self-focusing in nonlinear arrays of coupled waveguides, Optics Letters, Vol.13, PP. 794- 796, 1988.

[2] Lederer, F.S. Darmanyan, A. Kobyalov, Discrete solitons, in Trillo, S, Toruellas, WE (Eds) Spatial solitons. Springer-Verlag. Berlin, 2001.

[3] J.R. Bogning, A.S. Tchakoutio Nguetcho, T.C. Kofané, Gap solitons coexisting with bright soliton in nonlinear fiber arrays. International Journal of nonlinear science and numerical simulation, Vol. 6(4), PP. 371-385 , 2005.

[4] D. Cai, A.R. Bishop, N. Gronbech-Jensen, Localized states in discrete nonlinear Schrödinger equations, Physical review Letters. Vol. 72,PP. 591- 595 , 1994.

[5] E.W. Laedke, K. H. Spatschek, S.K. Turitsyn, Stability of discrete solitons and quasi collapse to intrinsically localized modes. Physical Review Letters, Vol. 73, PP. 1055-1058, 1994.

[6] M. J. Wadati, Wave propagation in nonlinear lattice. Journal of Physical Society Japan, Vol. 38, PP. 673- 680, 1975.

[7] M.J. Wadati, Wave propagation in nonlinear lattice. II. Journal of Physical society Japan Vol. 38, PP. 681- 686, 1975.

[8] K. Konno, M.J. Wadati, Simple derivative of Bäcklund transformation from Ricatti form of inverse method. Progress of Theory Physics, Vol. 53, PP. 1652- 1656, 1975.

[9] R. Hirota, Exact solution of the korteweg-de Vries equation for multiple collisions of solitons. Physical Review Letters Vol. 27, PP. 1192- 1194, 1971.

[10] C.S. Gardner, J.M. Greene, M.D. Kruskal, R.M. Miura, Method for solving the Korteweg-deVries equation. Physical Review Letters Vol. 19 , PP. 1095-1097 , 1967.

[11] A. M. Wazwaz, The tanh and the sine-cosine methods for the complex modified KdV and the generalized KdV equations,

Appled Mathematics and Computations, Vol. 154, PP. 713-723, 2004.

[12] A.M. Wazwaz, The tanh method: Exact solutions of the sine-Gordon and the sinh-gordon equations. Applied Mathematics and Computations, Vol. 167, PP. 1196-1210 , 2005.

[13] E. Fan, Extended tanh-function method and its applications to nonlinear equations, Physics Letters Vol. A277, PP. 212-218 , 2000.

[14] E. Fan, Y.C. Hon, pplications of extended tanh method to "special" types of nonlinear equations, Applied Mathematics and Computations, Vol. 141,PP. 351-358, 2003.

[15] A.M. Wazwaz, The tanh-coth and the sine-cosine methods for kinks, solitons and periodic solutions for the pochhammer-Chree equations, Applied Mathematics and Computations, Vol. 195, PP. 24-33 , 2008.

[16] C.T. Djeumen Tchaho, J.R. Bogning, T.C. Kofané, Construction of the analytical solitary wave solutions of modified Kuramoto-Sivashinsky's equation by the method of identification of coefficients of the hyperbolic functions. Far East Journal of Dynamical Systems, Vol. 14(1), PP. 17-34 , 2010.

[17] C. T. Djeumen Tchaho, J. R. Bogning, T. C. Kofané, Multi-Soliton solutions of the modified Kuramoto-Sivashinsky's equation by the BDK method. Far East Journal of Dynamical Systems, Vol. 15(1), PP. 83-98, 2011.

[18] J.R. Bogning, C.T. Djeumen Tchaho, T.C. Kofané, Construction of the soliton solutions of the Ginzburg- Landau equations by the BDK method, Physica Scripta Vol. 85, PP. 025013-025018, 2012.

[19] Wazwaz, AM: A modified KdV-type equation that admits a variety of travelling wave solutions: Kinks, Solitons, Peakons and cuspons. Phys. Scr. 86, 045501- 045506 (2012)

[20] C.T. Djeumen Tchaho, J.R. Bogning , T.C. Kofané, Modulated Soliton Solution of the Modified Kuramoto-Sivashinsky's Equation, American Journal of Computational and Applied Mathematics, Vol. 2(5), PP. 218-224, 2012.

[21] Qian-Yong Chen, Panayotis, G. Kevrekidis, B. Malomed, Dynamics of bright solitons and soliton arrays in the nonlinear Schrödinger equation with a combination of random and harmonic potentials, Physica Scripta, Vol. T141, PP. 014001-014007 , 2012.

[22] T. Kaladze, S. Mahmood, Hafeez Ur-Rehman, Acoustic nonlinear periodic (Cnoidal) waves and solitons in pair-ion plasmas, Physica Scripta Vol. 86, PP. 035506-035514, 2012.

[23] S. Ali Shan, A. Mushtaq, Arbitrary dust ion acoustic soliton with streaming ions and superthermal electrons. Physica. Scripta, Vol. 86, PP. 035503-035511, 2012.

[24] Jiu-Ning han, Jun-Hua Luo, Guang-Xing Dong, Shao-Shan Zheng, Ya-Gong Nan, Physica Scripta, Vol. 86, PP. 035505-035513, 2012.

[25] Qichang Jiang, Yanli Su , Xuanmang Ji, Coupling effects of grey-grey separate spatial screening soliton pairs. Physica Scripta Vol. 86, PP. 035404-035407, 2012.

[26] T. Yajima, M. J. Wadati, Solitons in an unstable medium, Physics Society Japan ,Vol. 59, PP. 41- 47,1990.

[27] D.J. Kaup, Higher-order water-wave equation and method for solving it, Progress of theory Physics, Vol. 54, PP. 396- 408, 1975.

[28] D. J. Kaup, T.I. Lakoba, The squared eigenfunctions of the massive Thirring model in laboratory coordinates, Journal of Mathematical Physics, Vol.. (N. Y) 37, PP. 308- 323, 1996.

[29] D.J. Kaup, T.I. Lakoba, Variational method: How it can generate false instabilities, Journal of Mathematical Physics, Vol. (N. Y) 37, PP. 3442- 3462, 1996.

[30] Biao Li, Yong Chen, On exact solutions of the nonlinear schrödinger equations in optical fiber. Chaos, Solitons and Fractals Vol. 21, PP. 241- 247, 2004.

[31] Biao Li, Yong Chen, Hongqing Zhang, Explicit exact solutions for new general two-dimensional KdV-Burgers-type equations with nonlinear terms of any order, Journal of Physics A: Mathematical and general, Vol. 35, PP. 8253-8265,2002.

[32] Basanti Mandal, A. R. Chowdhury, Solitary optical pulse propagation in fused fiber coupler-effect of Raman scattering and Switching, Chaos, Solitons and Fractals , Vol. 24, PP. 557- 565, 2005.

[33] M.A. Bisyarin, I.A. Molotkov, Subpicosecond pulse propagation in optical fibres with transverse and longitudinal inhomogeneities, Chaos, Solitons and Fractals, Vol. 17, PP. 297- 304, 2003.

[34] K. Nakkeeran, bright and dark optical solitons in fiber media with higher order effects, Chaos, Solitons and Fractals Vol.17, PP. 673- 679, 2002.

[35] M. N. Vinoj, V.C. Kuriakose, K. Porsezian, Optical soliton with damping and chirping in fibre media, Chaos, Solitons and Fractals, Vol. 12, PP. 2569- 2575, 2001.

[36] J. R. Bogning, T. C. Kofané, Analytical solutions of the discrete nonlinear Schrödinger equation in arrays of optical fibers, Chaos, Solitons & Fractals , Vol. 28, PP. 148-153, 2006.

[37] Ken-ichi Maruno, Adrian Ankiewicz, Nail Akhmediev, exact localized and periodic solutions of the discrete complex Ginzburg-landau equation. Optics communications, Vol. 221, PP. 199- 209, 2003.

[38] J. M. Soto-Crespo, Nail Akhmediev, Adrian Ankiewicz, Motion and stability properties of solitons in discrete dissipative structures, Physics Letters, Vol. A 314, PP. 126-130, 2003.