

Mathematical Modeling of Processes of Electro-Magnetic Fields' Effectson Thin Conducting Plates by Complex Form

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Abstract In this work the mathematical model of processes of electro-magnetic fields' effects on thin conducting plates by complex form, calculating algorithm mining by the joint using variation method and analytical method RFM and software for this algorithm and so calculating experiment are given

Keywords Mathematical Modeling, Magneto-Elasticity, Thin Plates

1. Introduction

Constructions, machines and devices elements which are in electromagnetic fields of variousorigins are widely applied in modern technic, physics and electrical engineering. Particularly, such elements are found in the measuring devices systems, radio engineering, computer science. In most cases elements of such constructions are thinelec-tro-conductive plates, and for this reason research of elec-tromagnetic fields influence on them is in a great interest.

2. Problem Statement

The plate in the rectangular x_1, x_2, x_3 coordinate system is located so, that the median plane of a plate whichcoincides with x_1, x_2 coordinate plane has σ high electro-conductivity and is in an electro-magnetic field with a small perturbation. For the solution of this task the magneto-elasticity hypothe-sis of thin bodies is existedand according to tangential components of electric-field vector intensity driven in elec-tromagnetic field plate and normal components of elec-tric-field vector intensity driven in electromagnetic field plate to the plate thickness are constant. Analytically mag-neto-elasticity hypotheses for an internal problem are ex-pressed like that[1,2]:

$$u_1 = u - x_3 \frac{\partial w}{\partial x_1}, u_2 = v - x_3 \frac{\partial w}{\partial x_2}, u_3 = w(x_1, x_2, t), \quad (1)$$

$$e_1 = \phi(x_1, x_2, t), e_2 = \psi(x_1, x_2, t), h_3 = f(x_1, x_2, t),$$

here u, v, w - deflection of a plate median plane; ϕ, ψ, f - func-tions of driven electromagnetic field.

In such statement the resolving equations for thin plates in some special cases of an external magnetic field are as fol-lows.[1,2]:

$$\begin{aligned} D\Delta^2 w + 2\rho h \frac{\partial^2 w}{\partial t^2} &= P \\ &+ \frac{2h^3 \sigma}{3c} \left(B_{02} \frac{\partial^2 \phi}{\partial x_1^2} - B_{01} \frac{\partial^2 \phi}{\partial x_1 \partial x_2} + B_{02} \frac{\partial^2 \psi}{\partial x_1 \partial x_2} - B_{01} \frac{\partial^2 \psi}{\partial x_2^2} \right) + \\ &+ \frac{2h^3 \sigma}{3c^2} \frac{\partial}{\partial t} \left[\left(\frac{1}{\varepsilon \mu} B_{02}^2 + B_{03}^2 \right) \frac{\partial^2 w}{\partial x_1^2} + \left(\frac{1}{\varepsilon \mu} B_{01}^2 + B_{03}^2 \right) \frac{\partial^2 w}{\partial x_2^2} - \frac{2}{\varepsilon \mu} B_{01} B_{02} \frac{\partial^2 w}{\partial x_1 \partial x_2} \right] \\ &+ \frac{2h \sigma}{c} \left[B_{02} \left(\phi - \frac{B_{02}}{c} \frac{\partial w}{\partial t} \right) - B_{01} \left(\psi + \frac{B_{01}}{c} \frac{\partial w}{\partial t} \right) \right], \quad \left(\Delta = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} \right). \\ \Delta \phi - \frac{\varepsilon \mu}{c^2} \frac{\partial^2 \phi}{\partial t^2} - \frac{4\pi \sigma \mu}{c^2} \frac{\partial \phi}{\partial t} &= \\ - \frac{\mu}{c} \frac{\partial}{\partial t} \left[\frac{4\pi \sigma}{c^2} B_{02} \frac{\partial w}{\partial t} - \frac{\varepsilon \mu - 1}{\varepsilon \mu} \left(B_{02} \frac{\partial^2 w}{\partial x_1^2} - B_{01} \frac{\partial^2 w}{\partial x_1 \partial x_2} - \frac{\varepsilon \mu}{c^2} B_{02} \frac{\partial^2 w}{\partial t^2} \right) \right], \\ \Delta \psi - \frac{\varepsilon \mu}{c^2} \frac{\partial^2 \psi}{\partial t^2} - \frac{4\pi \sigma \mu}{c^2} \frac{\partial \psi}{\partial t} &= \\ \frac{\mu}{c} \frac{\partial}{\partial t} \left[\frac{4\pi \sigma}{c^2} B_{01} \frac{\partial w}{\partial t} + \frac{\varepsilon \mu - 1}{\varepsilon \mu} \left(B_{02} \frac{\partial^2 w}{\partial x_1 \partial x_2} - B_{01} \frac{\partial^2 w}{\partial x_2^2} + \frac{\varepsilon \mu}{c^2} B_{01} \frac{\partial^2 w}{\partial t^2} \right) \right], \\ D &= \frac{2Eh^3}{3(1-\nu^2)} \quad (2) \end{aligned}$$

here: ε -dielectric constant; μ -magnetic permeability coef-ficient; c - electro-dynamic constant numerically is equal to the light speed in vacuum; σ -electro-conductivity coeffi-cientof an environment; $B_0 (B_{01}, B_{02}, B_{03})$ - magnetic induc-tion vector for internal area; ρ - platinum material density ; P - external force of non-electromagnetic origin; E - Young model; ν - Poisson's ratio.

The equations of magneto-elasticity (2) are solvedat relevant boundary conditions, depended on plate edges fixation technique[1,2]:

Rigidly clamped edge of a plate

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$$w|_r=0, \phi|_r=0, \psi|_r=0, \frac{\partial w}{\partial n}|_r=0 \quad (3)$$

Simply supported edge of a plate

$$w|_r=0, M|_r= -D \left(\frac{\partial^2 w}{\partial n^2} + \nu \frac{\partial^2 w}{\partial \tau^2} \right) |_{r=0}, \phi|_r=0, \psi|_r=0, \quad (4)$$

where: n, τ - direction of external normal and tangent to Γ border of a plate and at the given initial conditions:

$$\phi = \phi_0, \varphi = \varphi_0, w = w_0, \quad \frac{\partial \phi}{\partial t} = \dot{\phi}_0, \frac{\partial \varphi}{\partial t} = \dot{\varphi}_0, \frac{\partial w}{\partial t} = \dot{w}_0, \quad \text{att}=0, \quad (5)$$

here: $\varphi_0, \psi_0, w_0, \phi'_0, \psi'_0, w'_0$ - accordingly, given at the initial moment of time, where $t=0$ the value of unknown functions and the changes of their speed is zero.

The problem of magneto-statics is solved at the first stage of the given problem to define unknown components of B_{0i} magnetic induction for plates magneto-elasticity calculation expressed as the follows:

$$\frac{\partial}{\partial x_1} \left(\eta \frac{\partial A}{\partial x_1} \right) + \frac{\partial}{\partial x_2} \left(\eta \frac{\partial A}{\partial x_2} \right) = \frac{4\pi}{c} j \quad (6)$$

where

$$B_{01} = \frac{\partial A}{\partial x_1}, B_{02} = \frac{\partial A}{\partial x_2}, B_{03} = A, \eta = \mu_0 \mu, \mu_0 = 1.$$

For the equation (6) conditions are defined [3,4]:

$$\frac{\partial A}{\partial x_1} \Big|_{x_1=\pm a} = 0, \quad \frac{\partial A}{\partial x_2} \Big|_{x_2=\pm b} = 0 \quad (7)$$

Substituting values of a magnetic induction into equation (2), defined by magneto-statics problem solving (6) - (7) is dynamical problem of thin plates magneto elasticity of complex form.

3. Solution Methods

The problem solution is carried out by Bubnov-Galerkin variation method. At the first stage there have been given the solution structure for the main edge conditions at plate's complex form by RFM method [5,6]. Let's demonstrate the solution structure of the basic boundary conditions (3), (4) and (7). The solution structures for conditions (7), Rigidly clamped edge and Simply supported edges are as follows:

$$1. A = \Phi_0 - \varpi \left(\frac{\partial \Phi_0}{\partial x} \frac{\partial w}{\partial x} + \frac{\partial \Phi_0}{\partial y} \frac{\partial w}{\partial y} \right) \quad (8)$$

$$2. w = \omega^2 \Phi_1, \phi = \omega \Phi_2, \varphi = \omega \Phi_3 \quad (9)$$

$$3. w = \omega \Phi_1 - \frac{1}{2} \varpi^2 \left[\Phi_1 (D_2 \varpi + \nu T_2 \varpi) + 2 D_1 \Phi_1 \right], \quad (10)$$

$$\phi = \omega \Phi_2, \varphi = \omega \Phi_3$$

where

$$\Phi_0 = \sum_{i+j=1}^{N_0} c_{ij}(t) \phi_i(x_1) \varphi_j(x_2), \quad \Phi_1 = \sum_{i+j=1}^{N_1} c_{ij}(t) \phi_i(x_1) \varphi_j(x_2),$$

$$\Phi_2 = \sum_{i+j=N_1+1}^{N_1+N_2} c_{ij}(t) \phi_i(x_1) \varphi_j(x_2),$$

$$\Phi_3 = \sum_{i+j=N_1+N_2+1}^{N_1+N_2+N_3} c_{ij}(t) \phi_i(x_1) \varphi_j(x_2)$$

Here ω - the normalized equation of plate border, ϕ_i, ψ_i - some full (basic) systems of functions (polynomial function,

Chebyshev, trigonometric function, etc.), c_{ij} - undetermined coefficient of solution structure, definable; D_1, D_2, T_2 - linear differential operators [5, 6].

Substituting (8), (9), (10) solution structures in the equation (6) and movements (2), having carried out digitization procedure on spatial variables we will receive resolving equations (discrete model) for a finding of undetermined coefficient of solution structure. In case of statics the resolving equation is represented by system of the linear algebraic equations (SLAE):

$$BC = F \quad (11)$$

In case of dynamics the resolving equation is represented by system of the ordinary differential equations (SODE) with initial conditions

$$M\ddot{C} + BC = F \quad (12)$$

With initial conditions

$$\dot{C} \Big|_{t=t_0} = \dot{C}_0, C \Big|_{t=t_0} = C_0 \quad (13)$$

where: M, B - accordingly matrixes of weights and rigidity; F - a vector of the right part; C - a vector of indeterminate components.

For the solution of the resolving equations (11) and (12)-(13) it is possible to use known numerical methods of algebra and the analysis, particularly, for the solution of (11) - Gaussian elimination method [7,8], and for the solution (12)-(13) - Newmark method [7,9]. Thus calculation of certain integrals representing components for weight and rigidity matrixes is quite applicable Gaussian double integrals calculation numerical method.

4. The Software

Appropriate software representing a program complex is created on the basis of the developed computing algorithm. The given program complex consists of several modules: for formation of the geometrical information, i.e. for the equations of geometry of areas, their derivatives; for formation of the analytical information, in particular for definition of sub integral expressions; for calculation of solution structure, i.e. for structural formulas; for formation of elements of matrixes of the resolving equations, in particular realization of Gaussian method and so on; formation of points and scales cubature formulas for calculation of integrals; realization of the basic constructive means of R-functions theory, particularly calculation of value of the basic R-operations, differential operators; for solution of the resolving equations, in particular, for the solution of SLAE - Gaussian elimination method, for the decision SODE - Newmark method, calculations of value of basic polynomial, for example power function, trigonometric, Chebyshev; for registration of results of calculation, i.e. construction of tables of decisions, schedules of isolines.

5. Computing Experiment

The difficult form of area of a plate is considered in the

form of a rectangle with the approximated corners.

The normalized equation of the given area is as follows [5,6]:

$$\begin{aligned} \omega &= (((f_1 A_0 f_2) V_0 (f_3 A_0 f_4)) V_0 f_5) V_0 f_6) V_0 f_7) V_0 f_8 \\ f_1: &= (a^2 - x_1^2)/2a > 0; f_2: = ((b-r)^2 - x_2^2)/2b > 0; f_3: = \\ &= (b^2 - x_2^2)/2b > 0; \\ f_4: &= ((a-r)^2 - x_1^2)/2a > 0; f_5: = (r^2 - (x_1 - (a-r))^2 - (x_2 - \\ &= (b-r))^2)/2r > 0; \\ f_6: &= (r^2 - (x_1 + (a-r))^2 - (x_2 - (b-r))^2)/2a > 0; f_7: = (r^2 - (x_1 \\ &+ (a-r))^2 - (x_2 + (b-r))^2)/2a > 0; \\ f_8: &= (r^2 - (x_1 - (a-r))^2 - (x_2 + (b-r))^2)/2a > 0. \end{aligned}$$

At carrying out of computing experiment as initial data at the solution of problems are chosen: $a=1$ m, $b=0.5$ m (the rectangle party), $r=0.1$ m (radius of a rounding off of corners of a plate) (rice 2.), $h=0.01$ m, $E=7.1 \cdot 10^{11}$ H/M² (the elasticity module), $\nu=0.3$ (Poisson ratio), $\mu=0.001$ Gn/m (magnetic permeability), $\sigma=5.3 \cdot 10^{17}$ (Ohm)⁻¹ (electroconductivity), $\varepsilon=1$ (dielectric permeability), $c=3 \cdot 10^8$ km/s; $j=0.01$ A/M², $\rho=8890$ kg/m³ (copper density).

Let's notice, that in the given work we will not stop on the problem solution magnetostatics since it has been in detail enough resulted in the work[11].

Results of numerical calculations are more low resulted. In tables 1-2 are resulted a plate bend at action only mechanical forces (fur) and at action of electromagnetic and mechanical forces in characteristic points (0; 0) and (0,5; 0,5) areas, under above resulted boundary conditions, during time moments t changes from 0 to T with step 0.4 sec, $T=4$ sec

Table 1. Plate deflections in the is Rigidly clamped boundary condition

t	$w^*(x_1, x_2, t)$			
	<i>mech</i>	<i>magn</i>	<i>mech</i>	<i>magn</i>
(x_1, x_2)	(0,5; 0,5)	(0,5; 0,5)	(0; 0)	(0; 0)
0,4	0,001436	0,001416	0,003763	0,003961
0,8	0,003498	0,003598	0,004353	0,004052
1,2	0,008726	0,008822	0,003020	0,003035
1,6	0,009205	0,009006	0,001734	0,009348
2	0,002423	0,002523	0,001230	0,002204
2,4	0,001369	0,001379	0,003474	0,004841
2,8	0,000744	0,000745	0,001726	0,001926
3,2	0,003744	0,001384	0,004039	0,000196
3,6	0,004246	0,004348	0,000393	0,000193
4	0,008445	0,008745	0,003758	0,002881

Table 2. Plate deflections in Simply supported boundary condition

t	$w^*(x_1, x_2, t)$			
	<i>mech</i>	<i>magn</i>	<i>mech</i>	<i>magn</i>
(x_1, x_2)	(0,5; 0,5)	(0,5; 0,5)	(0; 0)	(0; 0)
0,4	0,004739	0,001416	0,009339	0,003465
0,8	0,005785	0,001598	0,001127	0,009181
1,2	0,002987	0,002822	0,004875	0,002277
1,6	0,003451	0,009006	0,006265	0,002366
2	0,006448	0,000252	0,001284	0,007437
2,4	0,001121	0,001379	0,008736	0,003536
2,8	0,002116	0,000745	0,007288	0,002486
3,2	0,006665	0,001384	0,006334	0,003413
3,6	0,002307	0,004348	0,004020	0,009672
4	0,009565	0,008745	0,002669	0,002091

Analyzing the numerical results received at calculations

resulted in tables 1-2 it is possible to consider, that magnetic field action is observed owing to influence Lorentz's forces, and is considerable under Simply supported boundary condition.

Further numerical results of calculation a component of the induced electromagnetic field φ and ψ are resulted at a time interval $t = [0; 4]$; $\Delta t = 0,4$, accordingly in the same characteristic points of area (tab. 3-4).

Table 3. Components of the induced electromagnetic field

t	Rigidly clamped edge		Simply supported edge	
	$\varphi(0; 0) \cdot 10^3$	$\psi(0; 0) \cdot 10^3$	$\varphi(0; 0) \cdot 10^3$	$\psi(0; 0) \cdot 10^3$
0,8	-0,001647	0,002342	0,009886	0,001532
1,2	-0,001044	0,005848	-0,004954	-0,002337
1,6	-0,001900	0,001096	-0,004294	0,003012
2	0,000742	-0,009653	-0,002301	0,007080
2,4	0,003971	-0,004275	-0,002157	-0,002054
2,8	-0,001273	0,001538	0,007615	0,009546
3,2	-0,001418	0,002546	0,002834	0,002586
3,6	-0,002003	0,002413	-0,0005429	0,002042
4	0,005127	-0,001001	-0,0003167	-0,001475

Table 4. Components of the induced electromagnetic field

t	Rigidly clamped edge		Simply supported edge	
	$\varphi(0,5; 0,5) \cdot 10^3$	$\psi(0,5; 0,5) \cdot 10^3$	$\varphi(0,5; 0,5) \cdot 10^3$	$\psi(0,5; 0,5) \cdot 10^3$
0,8	-0,008011	0,007680	-0,000119	0,000108
1,2	-0,003958	0,004356	0,000169	-0,000165
1,6	-0,001917	0,002547	-0,000109	0,000111
2	0,006445	-0,006283	-0,000220	0,000219
2,4	0,005815	-0,005952	0,000322	-0,000314
2,8	-0,004554	0,004165	-0,000510	0,000473
3,2	-0,001110	0,001041	-0,000207	0,000207
3,6	-0,007164	0,007018	0,000206	-0,000206
4	0,007322	-0,007050	-0,000254	0,000171

Thus, it is possible to notice, that here magnetic field parameters varies very quickly in relation to a deflection w , have high frequencies and small amplitudes, a field component φ opposite varies under the relation a component ψ .

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