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Modified Kudryashov Method for Finding Exact Solutions of the (2+1)-Dimensional Modified Korteweg-De Vries Equation and Nonlinear Drinfeld-Sokolov System

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Abstract In this paper, the modified Kudryashov method (the rational Exp-function method) with the aid of symbolic computation has been applied to obtain exact solutions of the (2+1)-dimensional modified Korteweg-de Vries equations (mKdV) and nonlinear Drinfeld-Sokolov system. New exact solitary wave solutions are obtained with comparison of other solutions obtained before in literature.

Keywords (2+1)-Dimensional Korteweg-De Vries (Mkdv) Equation, Nonlinear Drinfeld-Sokolov System, The Modified Kudryashov Method, Solitary Solutions

1. Introduction

Nonlinear partial differential equations have an important place in the study of nonlinear optics, elasticity theory and plasma physics. As an important aspect of nonlinear science known of these are solitary waves.

In this paper, we going to find solitary wave solutions for the (2 + 1)-dimensional modified Korteweg-de Vries equations equation[1].

$$u_t + u_{xxx} - \frac{3u_x u_{xx}}{2u} + \frac{3u_x^3}{4u^2} + 2Av_x u + 2Au_x v = 0, \quad (1.1a)$$

$$u_x = v_y, \quad (1.1b)$$

with A being an arbitrary constant, Eqs. (1.1a) and (1.1b) studied using variable separation and nonlinear phenomena[1] and it possesses Painlevé property[2, 3].

Also, in this paper, we aim to cast light on the Drinfeld–Sokolov system which is given by

$$u_t + (v^2)_x = 0, \quad (1.2a)$$

$$v_t - av_{x,x,x} + 3bu_x v + 3cuv_x = 0, \quad (1.2b)$$

where a, b and c are constants. This system was introduced by Drinfeld and Sokolov as an example of a system of nonlinear equations possessing Lax pairs of a special form[4-6].

In this paper, we use the modified Kudryashov method (the rational Exp-function method)[7-9] to obtain new exact solitary wave solutions of the (2+1)-dimensional Korteweg-de Vries (mKdV) equation and the nonlinear Drinfeld-Sokolov system.

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2. The Modified Kudryashov Method

To illustrate the basic idea of the modified Kudryashov method, we first consider a general form of nonlinear equation

$$p(u, u_t, u_x, u_{xx}, u_{tt}, u_{xt}, \dots) = 0. \quad (2.1)$$

where P is a polynomial function with respect to the indicated variables or some function can be reduce to a polynomial function by using some transformation.

Making use of the travelling wave transformation

$$u = u(\xi), \quad \xi = \alpha(x - \beta t), \quad (2.2)$$

where α and β are arbitrary constants to be determined later. Then Eq. (2.1) reduces to a nonlinear ordinary differential equation (ODE)

$$p(u, -\alpha\beta u', \alpha u', \alpha^2 u'', \alpha^2\beta^2 u'', -\alpha^2\beta u'', \dots) = 0. \quad (2.3)$$

We shall seek a rational function type solution for a given partial differential equation, in terms of exp (ξ), of the following form

$$u(\xi) = \sum_{k=0}^{m} \frac{a_k}{[1+\exp(\xi)]^k}, \quad (2.4)$$

where $a_0, a_1, \dots \dots and\ a_m$ are constants to be determined to form the solution of (2.1).

We can determine m by balance the linear term of the highest order in (2.3) with the highest order nonlinear term.

Differentiating (2.4) with respect to ξ, introducing the result into Equation (2.3), and setting the coefficients of the same power of e^{ξ} equal to zero, we obtain algebraic equations. The rational function solution of the Equation (2.1) can be solved by obtaining $a_0, a_{1,\dots\dots,} a_m$ from this system[6].

3. Solutions of (2+1)-Dimensional Modified Korteweg-De Vries Equation

Eq. (1) can be rewritten as

$$4u^2u_t + 4u^2u_{xxx} - 6uu_xu_{xx} + 3u_x^3 + 8Au^2v_xu + 8Au^2u_xv = 0, \tag{3.1a}$$

$$u_x = v_y, \tag{3.1b}$$

By using the transformation

$$u(x,y,t) = U(\xi),\ v(x,y,t) = V(\xi),\ \xi = \alpha(x + \beta y - \gamma t), \tag{3.2}$$

where α, β and γ are arbitrary constant, then Eqs. (3.1a) and (3.1b) become

$$-4\gamma U^2U' + 4\alpha^2U^2U''' - 6\alpha^2UU'U'' + 3\alpha^2(U')^3 + 8AU^3V' + 8AU^2U'V = 0, \tag{3.3a}$$

$$U' = \beta V'. \tag{3.3b}$$

In order to determine values of m and n , we balance the linear term of the highest order partial derivative terms and the highest order nonlinear terms in Eq. (3.3a) and (3.3b) , then we get m=n=2.

By using the rational function in exp (ξ), we may choose the solutions of (3.3a) and (3.3b) in the form

$$U(\xi) = a_0 + \frac{a_1}{1+e^{\xi}} + \frac{a_2}{(1+e^{\xi})^2}, \tag{3.4a}$$

$$V(\xi) = b_0 + \frac{b_1}{1+e^{\xi}} + \frac{b_2}{(1+e^{\xi})^2}, \tag{3.4b}$$

where $a_0, a_1, a_2, b_0, b_1,$ and b_2 are arbitrary constants to be determined later

Differentiating (3.4a) and (3.4b) with respect to ξ introducing the result into equations (3.3a) and (3.3b), and setting the coefficients of the same power of e^{ξ} equal to zero, we obtain these algebraic equations

$$-4\gamma a_0^2 a_1 + 4\alpha^2 a_0^2 a_1 + 8A a_0^2 a_1 b_0 + 8A a_0^3 b_1 = 0,$$

$$16A a_0^2 a_2 b_0 + 32A a_0^2 a_1 b_1 + 16A a_0^3 b_2 - 8\gamma a_0 a_1^2 + 2\alpha^2 a_0 a_1^2 + 56A a_0^3 b_1 + 32\alpha^2 a_0^2 a_2 - 8\gamma a_0^2 a_2 - 28\gamma a_0^2 a_1 + 56A a_0^2 a_1 b_0 + 16A a_0 a_1^2 b_0 + 4\alpha^2 a_0^2 a_1 = 0,$$

$$8A a_1^3 b_0 + 96A a_0 a_1^2 b_0 - 48\gamma a_0^2 a_2 - 84\gamma a_0^2 a_1 - 48\gamma a_0 a_1^2 + 56A a_0^2 a_1 b_2 + 40A a_0^2 a_2 b_1 + \alpha^2 a_1^3 - 18\alpha^2 a_0 a_1^2 + 192A a_0^2 a_1 b_1 - 4\gamma a_1^3 + 96A a_0^2 a_2 b_0 + 96A a_0^3 b_2 + 36\alpha^2 a_0 a_1 a_2 + 48A a_0 a_1 a_2 b_0 + 168A a_0^3 b_1 + 168A a_0^2 a_1 b_0 - 36\alpha^2 a_0^2 a_1 - 24\gamma a_0 a_1 a_2 + 40A a_0 a_1^2 b_1 + 72\alpha^2 a_0^2 a_2 = 0,$$

$$16A a_1^3 b_1 + 280A a_0^3 b_1 + 240A a_0^3 b_2 + 480A a_0^2 a_1 b_1 + 280A a_0^2 a_1 b_2 + 200A a_0^2 a_2 b_1 + 64A a_0^2 a_2 b_2 + 200A a_0 a_1^2 b_1 + 64A a_0 a_1^2 b_2 + 96A a_0 a_1 a_2 b_1 - 100\alpha^2 a_0^2 a_1 - 24\alpha^2 a_0^2 a_2 - 84\alpha^2 a_0 a_1^2 + 16\alpha^2 a_0 a_2^2 + 16\alpha^2 a_1^2 a_2 + 240A a_0 a_1^2 b_0 + 280A a_0^2 a_1 b_0 - 140\gamma a_0^2 a_1 - 20\gamma a_1^3 + 240A a_0 a_1 a_2 b_0 + 240A a_0^2 a_2 b_0 + 32A a_0 a_2^2 b_0 + 32A a_1^2 a_2 b_0 + 40A a_1^3 b_0 - 7\alpha^2 a_1^3 - 120\gamma a_0^2 a_2 - 120\gamma a_0 a_1^2 - 16\gamma a_0 a_2^2 - 16\gamma a_1^2 a_2 - 120\gamma a_0 a_1 a_2 - 12\alpha^2 a_0 a_1 a_2 = 0,$$

$$64A a_1^3 b_1 + 24A a_1^3 b_2 + 280A a_0^3 b_1 + 320A a_0^3 b_2 + 640A a_0^2 a_1 b_1 + 560A a_0^2 a_1 b_2 + 400A a_0^2 a_2 b_1 + 256A a_0^2 a_2 b_2 + 400A a_0 a_1^2 b_1 + 256A a_0 a_1^2 b_2 + 56A a_0 a_2^2 b_1 + 56A a_1^2 a_2 b_1 + 384A a_0 a_1 a_2 b_1 + 144A a_0 a_1 a_2 b_2 - 100\alpha^2 a_0^2 a_1 - 176\alpha^2 a_0^2 a_2 - 116\alpha^2 a_0 a_1^2 - 56\alpha^2 a_0 a_2^2 - 26\alpha^2 a_1^2 a_2 + 20\alpha^2 a_1 a_2^2 + 320A a_0 a_1^2 b_0 + 280A a_0^2 a_1 b_0 - 140\gamma a_0^2 a_1 - 40\gamma a_1^3 + 480A a_0 a_1 a_2 b_0 + 320A a_0^2 a_2 b_0 + 128A a_0 a_2^2 b_0 + 128A a_1^2 a_2 b_0 + 40A a_1 a_2^2 b_0 + 80A a_1^3 b_0 - 23\alpha^2 a_1^3 - 160\gamma a_0^2 a_2 - 160\gamma a_0 a_1^2 - 64\gamma a_0 a_2^2 - 64\gamma a_1^2 a_2 - 20\gamma a_1 a_2^2 - 240\gamma a_0 a_1 a_2 - 228\alpha^2 a_0 a_1 a_2 = 0,$$

$$64A a_1 a_2^2 b_1 + 16A a_2^3 b_0 + 8\alpha^2 a_2^3 + 96A a_1^3 b_1 + 72A a_1^3 b_2 + 168A a_0^3 b_1 + 240A a_0^3 b_2 + 480A a_0^2 a_1 b_1 + 560A a_0^2 a_1 b_2 + 400A a_0^2 a_2 b_1 + 384A a_0^2 a_2 b_2 + 400A a_0 a_1^2 b_1 + 384A a_0 a_1^2 b_2 + 168A a_0 a_2^2 b_1 + 80A a_0 a_2^2 b_2 + 168A a_1^2 a_2 b_1 + 80A a_1^2 a_2 b_2 + 576A a_0 a_1 a_2 b_1 + 432A a_0 a_1 a_2 b_2 - 36\alpha^2 a_0^2 a_1 - 144\alpha^2 a_0^2 a_2 - 54\alpha^2 a_0 a_1^2 - 144\alpha^2 a_0 a_2^2 - 84\alpha^2 a_1^2 a_2 - 60\alpha^2 a_1 a_2^2 + 240A a_0 a_1^2 b_0 + 168A a_0^2 a_1 b_0 - 84\gamma a_0^2 a_1 - 40\gamma a_1^3 + 480A a_0 a_1 a_2 b_0 + 240A a_0^2 a_2 b_0 + 192A a_0 a_2^2 b_0 + 192A a_1^2 a_2 b_0 + 120A a_1 a_2^2 b_0 + 80A a_1^3 b_0 - 17\alpha^2 a_1^3 - 120\gamma a_0^2 a_2 - 120\gamma a_0 a_1^2 - 96\gamma a_0 a_2^2 - 96\gamma a_1^2 a_2 - 60\gamma a_1 a_2^2 - 8\gamma a_2^3 - 240\gamma a_0 a_1 a_2 - 252\alpha^2 a_0 a_1 a_2 = 0,$$

$$128A a_1 a_2^2 b_1 + 32A a_2^3 b_0 - 32\alpha^2 a_2^3 + 64A a_1^3 b_1 + 72A a_1^3 b_2 + 24A a_2^3 b_1 + 56A a_0^3 b_1 + 96A a_0^3 b_2 + 88A a_1 a_2^2 b_2 + 192A a_0^2 a_1 b_1 + 280A a_0^2 a_1 b_2 + 200A a_0^2 a_2 b_1 + 256A a_0^2 a_2 b_2 + 200A a_0 a_1^2 b_1 + 256A a_0 a_1^2 b_2 + 168A a_0 a_2^2 b_1 + 160A a_0 a_2^2 b_2 + 168A a_1^2 a_2 b_1 + 160A a_1^2 a_2 b_2 + 384A a_0 a_1 a_2 b_1 + 432A a_0 a_1 a_2 b_2 + 4\alpha^2 a_0^2 a_1 - 24\alpha^2 a_0^2 a_2 + 6\alpha^2 a_0 a_1^2 - 56\alpha^2 a_0 a_2^2 - 26\alpha^2 a_1^2 a_2 - 60\alpha^2 a_1 a_2^2 + 96A a_0 a_1^2 b_0 + 56A a_0^2 a_1 b_0 - 28\gamma a_0^2 a_1 - 20\gamma a_1^3 + 240A a_0 a_1 a_2 b_0 + 96A a_0^2 a_2 b_0 + 128A a_0 a_2^2 b_0 + 128A a_1^2 a_2 b_0 + 120A a_1 a_2^2 b_0 + 40A a_1^3 b_0 + 2\alpha^2 a_1^3 - 48\gamma a_0^2 a_2 - 48\gamma a_0 a_1^2 - 64\gamma a_0 a_2^2 - 64\gamma a_1^2 a_2 - 60\gamma a_1 a_2^2 - 16\gamma a_2^3 - 120\gamma a_0 a_1 a_2 - 48\alpha^2 a_0 a_1 a_2 = 0,$$

$$
\begin{aligned}
&64 A a_1 a_2^2 b_1 + 16 A a_2^3 b_0 + 8\alpha^2 a_2^3 + 16 A a_1^3 b_1 + 24 A a_1^3 b_2 \\
&\quad + 24 A a_2^3 b_1 + 32 A a_2^3 b_2 + 8 A a_0^3 b_1 + 16 A a_0^3 b_2 + 88 A a_1 \\
&\quad a_2^2 b_2 + 32 A a_0^2 a_1 b_1 + 56 A a_0^2 a_1 b_2 + 40 A a_0^2 a_2 b_1 + 64 A \\
&\quad a_0^2 a_2 b_2 + 40 A a_0 a_1^2 b_1 + 64 A a_0 a_1^2 b_2 + 56 A a_0 a_2^2 b_1 \\
&\quad + 80 A a_0 a_2^2 b_2 + 56 A a_1^2 a_2 b_1 + 80 A a_1^2 a_2 b_2 \\
&\quad + 96 A a_0 a_1 a_2 b_1 + 144 A a_0 a_1 a_2 b_2 + 4\alpha^2 a_0^2 a_1 + 8\alpha^2 \\
&\quad a_0^2 a_2 + 8\alpha^2 a_0 a_1^2 + 16\alpha^2 a_0 a_2^2 + 16\alpha^2 a_1^2 a_2 + 20\alpha^2 a_1 a_2^2 \\
&\quad + 16 A a_0 a_1^2 b_0 + 8 A a_0^2 a_1 b_0 - 4\gamma a_0^2 a_1 - 4\gamma a_1^3 \\
&\quad + 48 A a_0 a_1 a_2 b_0 + 16 A a_0^2 a_2 b_0 + 32 A a_0 a_2^2 b_0 + 32 A \\
&\quad a_1^2 a_2 b_0 + 40 A a_1 a_2^2 b_0 + 8 A a_1^3 b_0 + 4\alpha^2 a_1^3 - 8\gamma a_0^2 a_2 \\
&\quad - 8\gamma a_0 a_1^2 - 16\gamma a_0 a_2^2 - 16\gamma a_1^2 a_2 - 20\gamma a_1 a_2^2 - 8\gamma a_2^3 \\
&\quad - 24\gamma a_0 a_1 a_2 + 24\alpha^2 a_0 a_1 a_2 = 0,
\end{aligned}
$$

$$-a_1 + 2\beta b_2 - 2 a_2 + \beta b_1 = 0,$$

$$\beta b_1 - a_1 = 0. \tag{3.5}$$

Solving the system of algebraic equations (3.5) with the aid of Maple, we obtain two cases of solutions

Case 1

$$a_0 = 0,\ a_1 = \frac{3}{2}\frac{\beta\alpha^2}{A},\ a_2 = \frac{-3}{2}\frac{\beta\alpha^2}{A},$$
$$b_0 = \frac{-1}{8}\frac{\beta(\alpha^2-4\gamma)}{A},\quad b_1 = \frac{3}{2}\frac{\alpha^2}{A},\quad b_2 = \frac{-3}{2}\frac{\alpha^2}{A}, \tag{3.6}$$

By back substitution we get the following new exact solution for the (2+1)-dimensional mKdV equation

$$u_1(x,y,t) = \frac{3}{2}\frac{\beta\alpha^2}{A}\left[\frac{1}{1+\exp[\alpha(x+\beta y-\gamma t)]} - \frac{1}{(1+\exp[\alpha(x+\beta y-\gamma t)])^2}\right],$$

$$v_1(x,y,t) = \frac{-1}{2A}\left[\frac{1}{4}\alpha^2 - \gamma - \frac{3\alpha^2}{1+\exp[\alpha(x+\beta y-\gamma t)]} + \frac{3\alpha^2}{(1+\exp[\alpha(x+\beta y-\gamma t)])^2}\right], \tag{3.7}$$

Case 2

$$a_0 = \frac{-3}{8}\frac{\beta\alpha^2}{A},\ a_1 = \frac{3}{2}\frac{\beta\alpha^2}{A},\ a_2 = \frac{-3}{2}\frac{\beta\alpha^2}{A},$$
$$b_0 = \frac{-1}{8}\frac{(\alpha^2-4\gamma)}{A}, b_1 = \frac{3}{2}\frac{\alpha^2}{A},\quad b_2 = \frac{-3}{2}\frac{\alpha^2}{A}, \tag{3.8}$$

By back substitution new exact solution for the (2+1)-dimensional modified Korteweg-de Vries equation is obtained

$$u_2(x,y,t) = \frac{-3}{2}\frac{\beta\alpha^2}{A}\left[\frac{1}{4} - \frac{1}{1+\exp(\xi)} + \frac{1}{(1+\exp(\xi))^2}\right],$$

$$v_2(x,y,t) = \frac{-1}{2A}\left[\frac{1}{4}\alpha^2 - \gamma - \frac{3\alpha^2}{1+\exp(\xi)} + \frac{3\alpha^2}{(1+\exp(\xi))^2}\right],$$

where $\xi = \alpha(x+\beta y-\gamma t)$. (3.9)

4. Solutions for Nonlinear Drinfeld-Sokolov System

Let $u(x,t) = U(\xi),\ v(x,t) = V(\xi)$,
where $\xi = \alpha(x-\beta t)$ (4.1)

Eqs. (1.2a) and (1.2b) becomes

$$-\beta U' + 2VV' = 0, \tag{4.2a}$$
$$-\beta V' - a\alpha^2 V''' + 3bU'V + 3cUV' = 0. \tag{4.2b}$$

In order to determine values of m and n, we balance the highest order linear terms with the highest order nonlinear terms in Eqs. (4.2a) and (4.2b), then we get m=2 and n=1.

By using the rational function in exp (ξ), we may choose the solutions of (4.2a) and (4.2b) in the form

$$U(\xi) = a_0 + \frac{a_1}{1+e^{\xi}} + \frac{a_2}{(1+e^{\xi})^2}, \tag{4.3a}$$
$$V(\xi) = b_0 + \frac{b_1}{1+e^{\xi}}, \tag{4.3b}$$

where a_0, a_1, a_2, b_0 and b_1 are arbitrary constants to be determined. Differentiating (4.3a) and (4.3b) with respect to ξ introducing the result into equations (4.2a) and (4.2b), and setting the coefficients of the same power of e^{ξ} equal to zero, we obtain these algebraic equations

$$-a b_1\alpha^2 + 3 c b_1 a_0 + 3 b a_1 b_0 - b_1\beta = 0,$$

$$-b_1\beta + 3 c b_1 a_0 + 3 b a_1 b_0 + 6 b a_2 b_0 + 3 c b_1 a_1 - a b_1\alpha^2 + 3 c b_1 a_2 + 3 b a_1 b_1 + 6 b a_2 b_1 = 0,$$

$$6 c b_1 a_0 + 3 b a_1 b_1 + 6 b a_1 b_0 + 6 b a_2 b_0 + 4 a b_1\alpha^2 - 2 b_1\beta + 3 c b_1 a_1 = 0,$$

$$\beta a_1 - 2 b_1 b_0 = 0,$$

$$\beta a_1 - 2 b_1^2 + 2\beta a_2 - 2 b_1 b_0 = 0. \tag{4.4}$$

Solving the system of algebraic equations (4.4) with the aid of Maple, we obtain the solutions

$$a_0 = \frac{1}{6ca_1}(a_1^2 b - ca_1^2 - 8b_0^2),\quad a_1 = a_1,\qquad a_2 = -a_1,$$
$$b_0 = b_0,\quad b_1 = -2\,b_0,\quad \beta = \frac{-4b_0^2}{a_1},$$
$$\alpha = \pm\sqrt{\frac{-2a_1 b - a_1 c}{2a}}. \tag{4.5}$$

Inserting Eqs. (4.5) in to (4.3a) and (4.3b), we get the following solitary wave solutions of Nonlinear Drinfeld-Sokolov system as follows

$$u_{1,2}(x,t) = \frac{a_1^2 b - ca_1^2 - 8b_0^2}{6ca_1} + \frac{a_1}{1+\exp(\xi)} - \frac{a_1}{(1+\exp(\xi))^2},$$

$$v_{1,2}(x,t) = b_0 - \frac{2b_0}{1+\exp(\xi)},$$

where $\xi = \pm\sqrt{\frac{-2a_1 b - a_1 c}{2a}}\left(x + \frac{4b_0^2}{a_1}t\right)$ (4.6)

5. Conclusions

In this paper, we have applied the modified Kudryashov method (the rational Exp-function method) to obtain new solitary wave solutions of the (2+1)-dimensional modified Korteweg-de Vries equation, and Nonlinear Drinfeld-Sokolov system. The obtained solutions are new and it reflexes how the method is powerful and can be applied on other nonlinear models.

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