

# A Distributed Algorithm for Un-balanced Partitioning of a Swarm of Autonomous Mobile Robots and Its Performance Analysis

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**Abstract** This piece of work studies the partitioning problem on independently operating swarm of autonomous mobile robots and devises algorithms for unbalanced partitioning in a distributed computing environment. The robots considered here are all identical and are very simple and weak. There is no central control over the robots and the robots do not communicate among themselves. Each robot executes the same algorithm based on their local information. This paper frames the algorithms for unbalanced partitioning by sorting the robots based on their ranking and then allocating them in different groups based on their ranks, such that N robots are divided into K unbalanced groups of unequal robots in each group. This paper also presents the performance based analysis of the un-balanced algorithm U\_PART over the balanced algorithm and examines their effects via different examples through 50 separate test cases. It also tries to bring out the shortcomings of the proposed U\_PART algorithm and proposes another alternative approach towards un-balanced partitioning to overcome the limitation of the U\_PART algorithm.

**Keywords** Swarm Robots, Un-balanced Partitioning, Distributed Algorithm

## 1. Introduction

Swarm robotics is a new approach to the coordination of multi robots which consist of a large number of simple, identical, autonomous and mobile robots over a distributed computing environment[1,2,4]. Such systems mainly consider identical point robots that are oblivious and autonomous, with strong fault tolerance capabilities as a group[5]. These concepts of swarm robotics have various applications such as task allocation[6], military operations, search and rescue victims, lawn mowing and sweeping, space mission, operations like enclosing an invader[8], area exploration and coverage[7] etc. Existing task allocation techniques[6,9, 10] partition the swarm into several groups and dynamically allocate each group of robots to multiple tasks. They may use balanced partitioning techniques[1-3] that partitions n number of robots in k-size balanced groups. But such approach may take large amount of time to complete a job, when groups are allocated multiple tasks with different workloads, as discussed in an example.

Consider the problem, where swarm of robots is assigned to paint 50 doors and 50 windows of a building. The area of each window is  $(3*2)6$  square feet and area of each door is

$(6*3)18$  square feet respectively and the swarm contain a total of 10 robots.

Say, the whole swarm is partitioned into two groups. Group 1 and group 2 are assigned to colour the windows and doors respectively. As per balanced partitioning each group contains 5 robots. The time required to colour the doors will be greater than to colour the windows by five robots as the total area of the windows is much lesser than the total area of the doors. In this situation, some robots in group 1 will complete its job and remain idle while others in group 2 are still with a larger work load. If the swarm is partitioned in an unbalanced manner of unequal members in each group, then the groups may be allocated task according to the work load, that is, smaller group may be assigned for smaller part of the job (colouring the windows) and the larger group can be assigned for the larger part (colouring the doors). Then no robots will be idle and time required in completing the job will be lesser than that of balanced Partitioning.

An algorithm U\_Part[4] has been proposed, to partition N number of mobile robots into K number of groups with unequal members in each group. This technique enhances the overall performance of the swarm; as less number of robots will be idle and time required in completing the job will be lesser than that of balanced partitioning. An alternative approach is also presented in the paper as algorithm U\_PartII. This paper also studies the performance analysis of the un-balanced algorithm over the existing balanced partitioning techniques presented in[1-3].

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## 2. Related Work

Large amount of research work has been reported on partitioning problem[1-3]. In most of them the robot follows the basic model *Wait-Observe-Compute-Move* model. Depending on the type of the problem the basic model is modified and used to devise solutions. The algorithms consist of a sequence of computational cycles, which is defined by a sequence of “*Look*”, “*Compute*”, and “*Move*” steps:

**Look:** Whenever a robot becomes active, it observes and marks the positions of all other robots, asynchronously and independently from the other robots, based upon its local coordinate system.

**Compute:** Depending on the observations made in the previous step, the robot calculates its destination point based on its own position and the current locations of the other robots, etc.

**Move:** The robot moves towards the destination point which has been calculated in the compute step.

The paper[3] by Asaf Afrima and David Peleg, studies the problem  $\text{PARTITION}(n,k)$ , which partitions  $n$  robots in  $k$  size balanced groups, and examines the solvability of this problem on various computation model. The algorithm discussed in this paper is *PART*, *Part2*, *Part\_Disperse*, *Part\_Expand*, *Part\_Asym*, *Part\_Sig* and *Part\_Sig2*.

The paper[1,2] reviews and explores various modification in the basic model developed in[3] and discusses their effects on the solvability of the partitioning problem.

David Peleg et. al.[1,2] has considered only balanced partitioning techniques and has stated the basic conditions for partitioning. They are as follows:

1. In the *FSYNC* and *SSYNC* model, having common axes direction is a necessary and sufficient condition. In *ASYNC* model, having common axes direction is necessary, and one common axis orientation is sufficient condition for  $\text{Partition}(n, k)$  to be deterministically solvable for all values of  $n$  and  $k$ .

2. In the half-compass model, the partitioning problem is solvable in the *SSYNC* model.

4. In the no-compass model, for any timing model,  $\text{Partition}(n, k)$  is unsolvable for  $k > 1$ .

5. In the axes-only model, for any timing model,  $\text{Partition}(n, k)$  is unsolvable for some values of  $n$  and  $k$ .

6. In the direction-only model and for the *SSYNC* timing model,  $\text{Partition}(n, k)$  is solvable for all values of  $n$  and  $k$ .

So far, the research works on partitioning problem has considered only the balanced approach for partitioning the swarm. In various task allocation techniques, the task load may vary in different groups. In such an environment the balanced partitioning technique will allocate task to groups having equal number of robots in each group. In such a setting, the task load will be more in some groups, whereas in other groups some robots remain unused. As a solution, this paper introduces the unbalanced partitioning techniques.

Here, two different algorithms for unbalanced partitioning are presented where a swarm of  $n$  robots are partitioned into  $k$  unbalanced groups. The first algorithm *U\_PART* presents

and unbalanced partitioning technique which creates groups of unequal members of robots. A simulator is being developed to check the time difference in completing distributed workloads, while using balanced and unbalanced group partitioning techniques. Next a comparative performance analysis between balanced and unbalanced techniques[11] has been carried out through the simulator. An algorithm *U\_Part* limits the number of members in each group (except the last group) for all values of  $N$ . this limitation has been overcome in the second algorithm *U\_PartII*.

## 3. Models, Assumptions and Problem Definition

Before going to describe the algorithm, let us discuss the assumptions and models used, and introduce the terminologies used in this paper. Our problem is to partition  $n$  number of robots into  $k$  no of size unbalanced groups. These robots are initially deployed within the priori known region. Robots may occupy any position within the region. We assume that no two robots occupy the same position. The robots are assumed to have the following characteristics[7,12].

1. **Identical and Homogeneous** - All the robots are identical in all respect, especially, they have the same computational capability. All the robots are assumed to be point robots with unlimited visibility. However, we assume that each of them is having a sensing zone of radius  $T$  ( $T$  is small). By this we mean that if a robot is required to carry out some job for a particular position (collection of information about that position, or painting that position etc.), instead of actually reaching the position, that can be carried out from a distance of  $T$  also as shown in “figure 1”

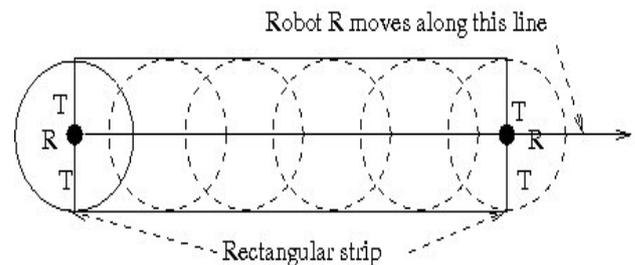


Figure 1. Sensing zone of radius  $T$

2. **Autonomous** - There is neither any central authority nor any external control over the robots. They do not even communicate among themselves.

3. **Mobile** - All robots are allowed to move on a plane.

4. **Computation Model** - Here we follow the basic *Observe-Compute-Move* model. A computational cycle is defined to be a sequence of “observe”, “compute” and “move” steps. Each of the robots executes same instructions in all the computational cycles. Once a robot completes one computational cycle, it starts executing the next one. The actions taken by a robot in *compute* and *move* steps, entirely depend on the observations made in *observe* step. In some situations, an observation might lead a robot not to change its position

in *move* step. In such cases the robot seems to be idle, though it is actually executing all the three steps.

**5. Oblivious or Memoryless** - Robots do not retain any information gathered in the previous computational cycle. In every computational cycle, a robot starts computing from very beginning depending only on the positions of the other robots observed at that computational cycle.

The robots can have two states: *active* and *sleep*. In *Active* state, the robots are alive and executing continuously the computational cycles. In *Sleep* state, robot is not active and doing nothing. This state is like “power off” state. It is assumed that a robot cannot *sleep* infinitely and it would become active within a finite amount of time. We also assume that change of state of a robot takes place independent of the other robots.

The operation considered here is assumed to be an *Atomic* operation. During the computation process, a robot cannot switch over to the “sleep” state also. The models considered here are as follows:

**Asynchronous model:** Robots operate on independent cycles of variable lengths. They do not share any common clock[7,12].

**Direction only:** Directions of both axes are common to all the robots, but the positive orientation of the axes may be different[7,12]. Here, we assume that x-axes of the robots are parallel to the known common reference line. Therefore, the direction of x-axis is common to all the robots but the robots may have different views of the positive orientation of the axis. However, it is assumed that the direction of the positive y-axis is 90° counter clockwise to the positive direction of the x-axis. Thus, direction of y-axis is also common to all the robots, except possibly the positive orientation. Each robot has its local co-ordinate system. All the robots would assume that they occupy the position (0, 0) with respect to their local co-ordinate system. Further, we assume that these various co-ordinate systems might not share a common scale. “Figure 2” shows the local co-ordinate systems of four robots R<sub>1</sub>, R<sub>2</sub>, R<sub>3</sub>, and R<sub>4</sub>, and the common reference line XX’.

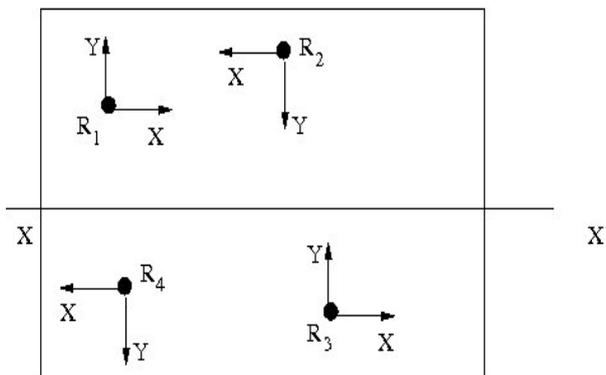


Figure 2. Local coordinate system of 4 robots

**Lemma 1:** The diagonals of a square intersects each other at the center of the square and makes an angle of 90° with each other.

**Lemma 2:** The angle traced by a complete circle is equal to 360°.

## 4. Partitioning Algorithm

The first part of this section describes the proposed algorithm for un-balanced partitioning. The correctness proves of the algorithm are given in the second part.

### 4.1. Algorithm for Unbalanced Partitioning

This research work proposed an algorithm to partition N number of robots into K number of unbalanced groups. Further, we assume that all N robots are enclosed within a known obstacle free square region. The robots need to be partitioned into K (Gr<sub>1</sub>, Gr<sub>2</sub>, Gr<sub>3</sub>, ..., Gr<sub>k</sub>) number of size-unbalanced groups. As the groups will not be balanced in size, so the total number of robots in each group always differs with the number of members in all other groups.

To create such unbalanced groups, each group will contain number of members equal to the group number except the last group which may or may not contain same number of robots equal to the group number depending on the value of N. As the group number and the number of members in a group are same, so now onwards value of group number is considered as the number of members in that group. Each robot contains a flag (FLAG) which is set with the value of the particular group number to which the robot is assigned.

As soon as a robot R becomes alive it performs the following computational cycle “*observe-compute-move*”. As long as a robot is in alive state, after completing one such computational cycle, it would again start another cycle and continue in this way until it completes its assigned job. Each robot in the swarm executes same algorithm U\_PART when active.

In the look state a robot gathers information of its neighbour. Then it computes its rank, group number and goal point. The rank is calculated with the help of the observed information. Using the rank of the robot its group number is calculated. The robot needs to move towards the respective goal points to form the partition. The number of goal points is same as the number of groups formed. So the goal point of robot R is calculated in the compute step. It moves to the goal point in the move step.

#### Algorithm U\_PART (Executed by Robot R)

##### Look:

According to the local co-ordinate system, a robot R first observes the position of all other robots. Let the co-ordinates be (a<sub>1</sub>, b<sub>1</sub>), (a<sub>2</sub>, b<sub>2</sub>), ..., (a<sub>n-1</sub>, b<sub>n-1</sub>), whereas, its own co-ordinate would be (0,0). It is to be noted here that some of these a<sub>i</sub>, b<sub>i</sub> values might be negative also.

##### Compute:

**Step 1:** According to the values of y-co-ordinates, the robot R will order all the robots (including itself) so that the robot having the largest value of y coordinate will have the highest rank, that is, N. Without loss of generality, we assume that the co-ordinates of N robots, after sorting are (x<sub>1</sub>, y<sub>1</sub>), (x<sub>2</sub>, y<sub>2</sub>), (x<sub>N</sub>, y<sub>N</sub>), so that (y<sub>1</sub> <= y<sub>2</sub> <= y<sub>3</sub> <= ... <= y<sub>N</sub>). The robot having the co-ordinate (x<sub>i</sub>, y<sub>i</sub>) would have the rank i and the robot will be mentioned as R<sub>i</sub>, 1 <= i <= N. In case of a tie, the values of x-coordinate of the robots are considered. The

robot having lower x-coordinate would have the lower rank.

In case of tie in the value of y-coordinate, means two robots are in a same location which is not possible practically. In this way, the robot R would determine its own rank. Let the rank of R be p. From now onwards R and  $R_p$  will be used interchangeably.

**Step 2:** The value of “K” that is; the total number of groups formed should be  $K \leq \text{ceiling}(N/2)$  and must be selected in such a manner that no two groups contain equal number of robots.

**Step 3:** Once a robot gets its rank it checks whether it is having the lowest or highest rank among all other robots. The lowest ranked robot always belongs to the first group and the highest ranked robots belong to the last group.

**Step 4:** If the robot is having rank “p” which is neither lowest nor highest, it checks whether  $p \leq 1 + Gr_1$ , if true it then sets the FLAG=2. If false it again checks  $p \leq 1 + Gr_2 + Gr_3$ , if true it sets FLAG=3. For each false outcome the rank is checked against the previous cumulative sum added to the next group number. This checking continues till  $p \leq (K(K-1))/2$ . If true, then FLAG=(K-1). If false then robot will be assigned to the Kth group.

**Step 5:** The robot R computes the group number to which it is allocated. It then needs to compute the goal point towards which it must move to form the partition. The robot R calculates the center of the square field which is the intersection of the diagonals. Now the robot R first checks its group number. If it belongs to group 1 then, it computes its goal point by calculating an angle  $\Theta$  (theta) where  $\Theta = 45^\circ$  from the center point in anti-clockwise direction along the line XX' and marks its goal point at a fixed distance “d” from the center, considering  $d \leq a/4$  where “a” is the dimension of the side of the square. If the robot R belongs to group 2, it then computes its goal point at an angle  $2\Theta$  at a distance “d” from the center. In this way the value of  $\Theta$  increases with the group number till  $\Theta$  reaches  $360^\circ$  or  $8\Theta$ .

If the group number exceeds  $8(Gr_K > 8)$  then the robot R computes the goal point at an angle  $\Theta = 45^\circ$  at a distance  $2d$  from the center. The angle  $\Theta$  increases in the same order as before and continues till it completes  $360^\circ$  or  $8\Theta$  for group number 16 in the second round. In this manner the goal point is calculated over the square field based on the group numbers to which a robot belongs. After completing each round of  $360^\circ$  the distance increases by “d” and the angle begins from  $\Theta$  every time.

To maintain the relative ranking throughout the process, the robot R may need to take a halt before reaching its final destination. In this compute step, the robot R should verify this situation and if required, it would recalculate the position of the halt. We call this as the secondary destination.

The “compute” step terminates as soon as the robot computes its destination, final or secondary. The “Figure 3” below shows the different goal points of the groups.

#### Move:

After identifying the goal point, the Robot R starts moving towards the goal point. On the way towards their destination, robots would maintain their relative ranking. It means, while

moving, robots should not cross vertically any other robot even if their routes do not intersect each other. In other words, to reach the destination, if a robot is going to gain a vertical height higher (lower) than a robot of higher (lower) rank (that is, it is crossing another robot which would affect the relative ranking), it would stop at an  $\epsilon$  (pre-defined small quantity) distance from that height and would wait for that other robot to move on. In this situation, the robot R may need to take a halt before reaching its final destination and if required it will recalculate the position of next halt. This halts are known as *secondary destination*.

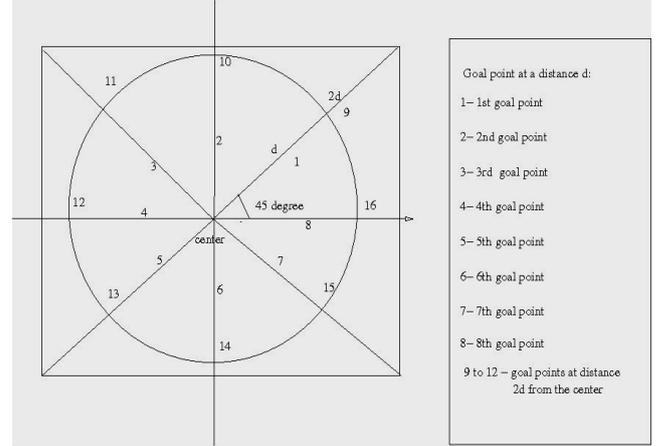


Figure 3. Different goal points for different robots

A robot would always move in vertical direction first, after acquiring the vertical height of the final destination, the robot would then move into horizontal direction to reach the final destination. Thus, to reach the secondary destination, a robot moves only in vertical direction. Depending on whether a robot reaches its final or secondary destination, the following two courses of actions would be taken by the robot:

(i) As soon as, a robot reaches the secondary destination, this “move” state terminates. That is, the computational cycle will be terminated and the robot will again start a new computational cycle with “observe” state.

(ii) Once the robot reaches its final destination that means the job is completed successfully, without any interruption. At the end, it would generate a signal that its job is done. At any point of time, if the robot R finds another robot R' at the same vertical height (which might occur at the starting time, if initially they are at the same height), then depending on the rank of R' and that of itself, R decides its next course of action as follows:

**Case A:** The rank of R is greater than that of R' and the destination of R is in the positive direction, w.r.t. its local co-ordinate system.

**Case B:** The rank of R is less than that of R' and the destination of R is in the negative direction, w.r.t. its local co-ordinate system.

For both the cases, R would break the tie and would move first towards its destination point.

**Case C:** The rank of R is greater than that of R' and the destination of R is in the negative direction, w.r.t. its local co-ordinate system.

**Case D:** The rank of R is less than that of R' and the destination of R is in the positive direction, w.r.t. its local co-ordinate system.

For both the cases R will wait for R' to move first towards its destination point.

**4.2. Correctness Proofs**

**4.2.1. Observation 1: The Overall Process Starts at a Finite Amount of Time**

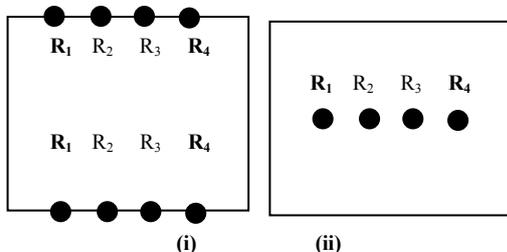
*Proof:* As mentioned before, the robots can have two states: active and sleep. In Active state, the robots are alive and executing continuously the computational cycles. In Sleep state, robot is not active and doing nothing. This state is like power off state. It is assumed that a robot cannot sleep infinitely and it would become active within a finite amount of time. We also assume that change of state of a robot takes place independent of the other robots. Thus whenever a robot becomes active it executes the algorithm and it is also assumed that the other robots will also become active after a finite time and will execute the same algorithm independently. Hence it is guaranteed that overall process starts at a finite time.

Only in case of a tie, if initially the robots are at the same height, there will be inter-dependency among these robots. If a particular robot do not move first all other have to wait for it and so on. The robot having the highest rank and the lowest rank usually does not have any restriction on their movement and thus as soon as they become live, the process starts. An extreme situation is considered when all the robots are at the same height.

The situation can be sub-divided into following two cases:

Case 1: All the robots are at the same height and they are along a boundary of the region. In this case, if a robot identify itself (according to its local co-ordinate system) (1) at the lower boundary of the region and having the highest rank, or (2) at the upper boundary of the region and having the lowest rank.

In both of the cases the robots will not have any restriction on its movement and it would break the initial barrier. These robots are called tie-breaking robots. If the robots are on the upper boundary, the left-most one and if they are on the lower boundary, the right-most one will be the tie-breaking robot.



**Figure 4.** R<sub>1</sub> and R<sub>4</sub> are the tie-breaking robots in two cases

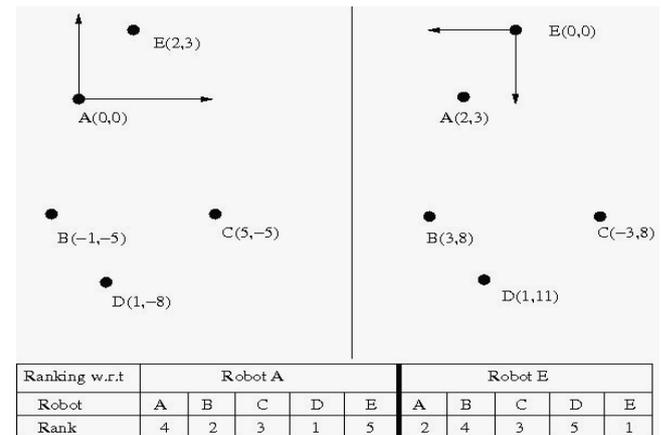
Case 2: All the robots are at a same height from the common reference line but they are not along any boundary of the region. In this case both the robots having lowest rank

and highest rank will not have any restriction on their movement and they would break the initial barrier. “Figure 4”, below shows both the cases where the tie-breaking robots either having highest or lowest rank. Once a robot break the initial barrier, all other robots start moving in turn. Thus within finite time the process would start.

**4.2.2. Observation 2**

Throughout the process, relative ranking of the robots computed by several robots are same up to a reversal of order. In other words, if the robots R<sub>1</sub> and R<sub>2</sub> compute the rank of a robot R as i and j respectively, then either i=j or i= N-j and this would remain same throughout the algorithm.

*Proof:* If the orientations of the local axes of R<sub>1</sub> and R<sub>2</sub> are identical then the ranking of the robots would be same. Otherwise, if the orientations are reverse, then the relative ranking would be same but in reverse order. Thus i=N-j. “Figure 5” shows the positions of five robots and their ranks with respect to robots A and E. Robots A and E are having opposite orientations. The ranks of the robots w.r.t A are just the reverse of the ranks w.r.t E.



**Figure 5.** Relative ranks w.r.t robot A and E are same up to a reversal of order

A robot computes the ranks of all other robots w.r.t. their vertical distances from its local x-axis. So the relative ranking of the robots would remain same throughout the algorithm as the vertical movement of the robots is so restricted that none of the robots would vertically cross any other robot.

If two robots are starting from the same vertical height, their relative ranking will be determined by their x- coordinates.

In case of such a tie, the robots start moving towards their destination following the rule given in “move” step, which retains their relative ranking.

Once a robot starts moving, this tie will be broken and this situation will never occur again.

**4.2.3. Observation 3: The Group Number Allocated to the Robots Remains Same Throughout the Process**

*Proof:* Robot R is allotted its group number according to its rank. Thus group 1 contains 1 robot, group 2 contains 2

robots of rank 2 and 3, and in this way  $(K-1)$  group contains “ $K-1$ ” number of robots. Thus remaining robots  $(N-(1+2+3+\dots+K-1))=N-(K(K-1))/2$  is contained in the  $K^{\text{th}}$  group and flag assigned is  $Gr_K$ . As the relative ranking is preserved and does not change throughout the algorithm henceforth the group number is preserved.

Example: Let us consider 6 robots, with relative ranking 1, 2, and 3, 4, 5, 6. Now the group numbers allocated will be as follows: Robot with rank 1 is assigned group 1, robots with ranks 2, 3 is assigned group 2 and robots with ranks 4, 5, 6 are assigned group 3. Thus if the relative ranking remains unchanged throughout the process then the group numbers allocated will certainly remain fixed throughout the process.

#### 4.2.4. Observation 4: $\Theta$ is Always Equal to $45^\circ$ and $8\Theta$ Completes a Circle Around the Center Point

*Proof:* The robot R calculates the center of the square field which is the intersection of the diagonals. Now the robot R first checks its group number. If it belongs to group 1 then, it computes its goal point by calculating an angle  $\Theta$  ( $\Theta=45^\circ$ ) from the center point in anti-clockwise direction and marks its goal point at a fixed distance  $d$  from the center, considering  $d \leq a/4$  where “ $a$ ” is the dimension of the side of the square.

Since the field is square in shape, by Lemma 1 above the angle made at the center by the diagonals are  $90^\circ$ . The center line parallel to x-axis passing through the center of the square field makes an angle of  $45^\circ$  with the diagonals. Thus  $\Theta$  is taken as  $45^\circ$ . Now the goal point is calculated at an angle  $\Theta$  at a distance  $d$  from the center if robot R belongs to group 1, considering  $d \leq a/4$  where “ $a$ ” is the dimension of the side of the square. If robot R belongs to group 2 the goal point is calculated at an angle  $2\Theta$  from the center at a distance  $d$  from the center. In this way the angle increases linearly as the group number increases till  $8\Theta$ . If the group number crosses 8, then the next angle again starts from  $\Theta$  and the goal point is calculated at a distance  $2d$  from the center and continues till it reaches  $8\Theta$ . After every round of  $8\Theta$  the next goal point is calculated at an angle  $\Theta$  and the distance from the center is incremented by  $d$ . By Lemma 2  $8 \times 45^\circ = 360^\circ$  completes one circle around the center point. Therefore the group numbers belonging to 8, 16, 32, 64 which consist of a GP series make an angle of  $8\Theta$  every time on completing  $360^\circ$  and the next group numbers belonging to 9, 17, 33, 65 starts calculating the goal point from the center of the field along the line passing through the center at an angle of  $\Theta$ .

#### 4.2.5. Observation 5: The Partitioning Algorithm Results an Unbalanced Groups of Robots

*Proof:* In the algorithm U\_PART above the robots are grouped according to their ranks. Thus group 1 contains 1 robot, group 2 contains 2 robots of ranks 2 and 3, and in this way  $(K-1)$  group contains  $K-1$  number of robots. Thus remaining robots  $(N-(1+2+3+\dots+K-1))=N-(K(K-1))/2$  is contained in the  $K^{\text{th}}$  group and flag assigned is  $Gr_K$ . Let us consider two cases as follows:

Case 1: Let 10 robots be present in the field. Then by the algorithm U\_PART, the 10 robots ranked from 1 to 10, by unbalanced partitioning robot  $R_1$  is assigned group  $Gr_1$ ,  $R_2$  and  $R_3$  are assigned  $Gr_2$ ,  $R_4, R_5, R_6$  is assigned  $Gr_3$  and robots  $R_7, R_8, R_9$  and  $R_{10}$  is assigned  $Gr_4$ .

The algorithm works correctly if the formula  $(N-(1+2+3+\dots+K-1))=N-(K(K-1))/2$  gives the same outcome. Verifying the formula:  $(n-(1+2+3+\dots+k-1))=n-(k(k-1))$ . For  $N=10$  and selecting  $K=4$ , since  $K \leq \text{ceiling}(N/2) = (10/2) = 5$ , therefore  $N-(K(K-1))/2 = 10 - (4(4-1))/2 = 10 - (4 \times 3)/2 = 10 - 6 = 4$ . Therefore there must be 4 robots present in the  $K^{\text{th}}$  group. As the outcome matches with the result obtained from the formula, the algorithm U\_PART works correctly.

Case 2: Now, considering for  $N=11$  robots,  $K=4$ , since  $K \leq \text{ceiling}(N/2)$ . Now,  $N-(K(K-1))/2 = 11 - (4 \times 3)/2 = 11 - 6 = 5$ . Therefore there are 5 robots present in the last group or group 4 in this case. Thus each group will contain number of members equal to the group number except the last group which may or may not contain same number of robots equal to the group number depending on the value of  $N$ , therefore  $Gr_1$  will contain 1 robot,  $Gr_2$  will contain 2 robots,  $\dots, Gr_K$  will contain  $[(2N-K(K-1))/2]$ , where unbalanced partition is guaranteed.

Henceforth it must also be kept in mind that the group number is so selected that no two groups have equal number of robots. Therefore unbalance partitioning is guaranteed.

#### 4.2.6. Observation 6: The Movement of the Robots is Collision Free

*Proof:* Throughout the algorithm, two robots can never be at the same vertical height at the same time, except during the starting time, when two robots can be at the same vertical height. If initially the robots are at the same height, the tie is broken by the rules given in Move step. Once the tie is broken, they will never be at the same height again, during their vertical movement.

After computing the destination, each robot would first move vertically to reach the height of the final destination. Once they reach that height, they start moving horizontally. Thus, if the destinations of the two robots are at different heights, the question of collision during their horizontal movements does not arise at all.

#### 4.2.7. Observation 7: The Four Rules Stated in the Move Step are Valid

*Proof:* If at the initial stage, two robots are at the same height (but in two different positions), the robot having the higher rank would start moving first, if their destinations are in the negative direction, then the robot having the lower rank would start moving first. If their destinations are in opposite direction, then there wouldn't be any restriction in the vertical movement. Consider “figure 6” below where both  $R_1$  and  $R_2$  have the same orientations. The destinations of robots  $R_1$  and  $R_2$  are  $d_1$  and  $d_2$  respectively. According to both the robots the rank of  $R_1$  is less than the rank of robot  $R_2$ . In the “figure 6(a)” both of their destinations are in the

positive direction, then as per the rule, the higher ranked robot  $R_2$  will move first. In “figure 6(b)” both of their destinations are in the negative direction. As per rule, the lower ranked robot  $R_1$  will move first.

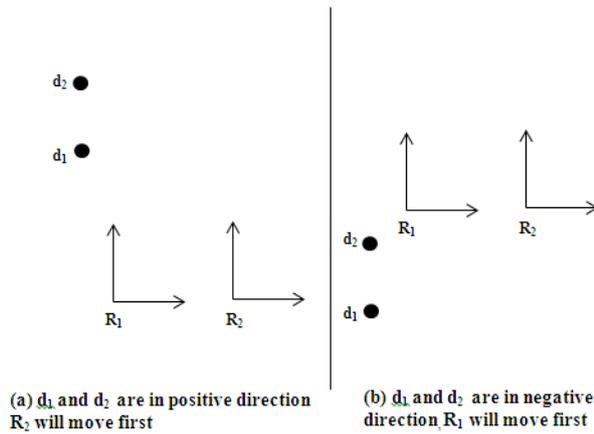


Figure 6.  $R_1$  and  $R_2$  are having same orientation

Consider “figure 7” where both  $R_1$  and  $R_2$  having opposite orientation. According to the local coordinate system of  $R_1$  and  $R_2$ , both will rank itself as lower. Due to opposite orientations, if the destinations are in positive direction according to  $R_1$  then it is in negative direction according to  $R_2$  and vice versa.

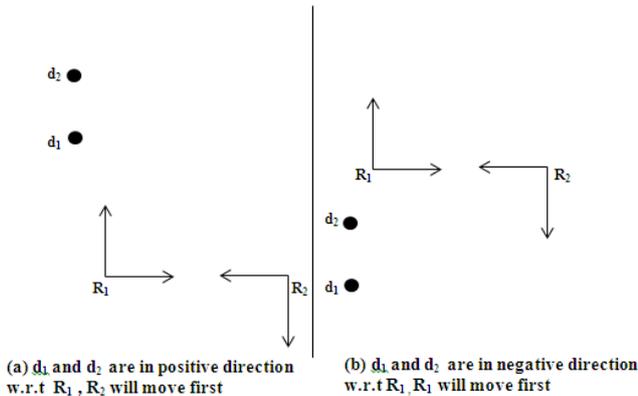


Figure 7.  $R_1$  and  $R_2$  are having opposite orientation

In “figure 7(a)” both the destinations are in the positive direction w.r.t  $R_1$ . So, according to  $R_1$ , the higher ranked robot  $R_2$  will move first. But according to  $R_2$  the destinations are in negative direction, so as per rule the lower ranked robot  $R_2$  will move first. This shows that the same decision will be taken by  $R_1$  and  $R_2$ . Similarly, in “figure 7(b)”, both the destinations are in negative direction w.r.t  $R_1$ . So, according to  $R_1$ , the lower ranked robot  $R_1$  will move first. But according to  $R_2$  the destinations are in positive direction, so as per rule higher ranked robot will move first which is  $R_1$  according to the local coordinate system of  $R_2$ . So, in both the cases same robot will move, and the tie will be broken without any conflict.

4.2.8. Result: The Partitioning Algorithm U\_PART Will Be Completed Within a Finite Amount of Time

*Proof:*

Combining all the above observations, and the fact that a robot cannot sleep for infinite amount of time and considering the operation to be atomic, we conclude that the overall process completes successfully within finite amount of time.

## 5. Simulation and Performance Analysis of Algorithm U\_PART

### 5.1. Simulation

In this section, the two partitioning techniques has been compared and analysed. An example of an academic building has been considered. The building is assumed to have a number of lecture rooms. Different number of chairs is placed in each room. A swarm of robots have been allocated to colour all the chairs placed in different rooms of the building. Colouring all the chairs in one room is considered as one task. Both the algorithms suggest partitioning the robot swarm into number of groups so that each group can complete at most one task. Assuming that numbers of groups are equal to the number of task to be done.

A simulator has been developed to evaluate and compare the overall performance of un-balanced algorithm over the balanced algorithm. 50 test cases have been analysed using the simulator to study the performance of the un-balanced algorithm over balanced algorithm. The simulator calculates the total time required to complete the whole job (painting all the chairs in all the rooms of the building) by the swarm by applying the balanced algorithm as well as the un-balanced algorithm. Based on the comparison of time required to complete the job the simulator suggests the technique to be used for a particular task to be done in minimum time.

### 5.2. System Requirements of the Simulator

#### 5.2.1. Software:

Turbo C++ version 4.5 compiler, Matlab 7, Windows XP Professional (SP2)

#### 5.2.2. Hardware

Processor: Pentium IV 1.4 GHz minimum, Disk Space: 2 GB, RAM: 1024 MB, DVD ROM drive, High resolution monitor (XGA recommended), Keyboard and Mouse.

### 5.3. Performance Analysis

The simulator studied 50 test cases to evaluate the performance of the unbalanced algorithm over the balanced algorithm. A swarm is allocated to colour all the chairs situated in different rooms inside the building. Colouring all the chairs in one room is considered as one task. It is assumed that one robot can paint a single chair within 5 minutes. The task is carried out concurrently by individual robot.

$N$  = Number of robots in the swarm.

$K$  = Number of groups formed.

$C_i$  = Number of chairs present in the  $i^{th}$  room.

$T_{ib}$  = Time taken to colour all the chairs in the  $i^{th}$  room, in minutes by balanced partitioning.

$T_{iub}$  = Time taken to colour all the chairs in the  $i^{th}$  room, in minutes by un-balanced partitioning.

TOB = Maximum time taken to complete colouring all the chairs present inside the building via balanced partitioning.

TOUB = Maximum time taken to complete colouring all the chairs present inside the building via un-balanced partitioning.

The tables in Appendix A (table no A.1& A.2) shows the results obtained from 50 test cases. Out of 50 test cases mentioned above, 3 random cases have been plotted to bring out the comparison between balanced and un-balanced techniques.

Problem 1: The building has two rooms, having 10 and 160 chairs respectively. The swarm is partitioned into two groups (one group per task) using balanced as well as un-balanced partitioning. Then the total time required to complete the task is compared and analysed for both partitioning techniques.

Problem 2: The building has three rooms, having 2, 50 and 300 chairs respectively. The swarm is partitioned into three groups using balanced as well as unbalanced partitioning. Then the total time required to complete the task is compared and analysed for both partitioning techniques.

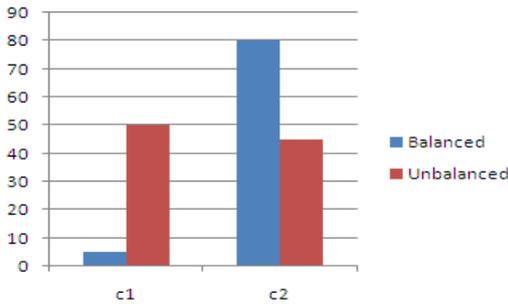


Figure 8. Represents problem 1

Problem 3: The building has four rooms, having 4, 40, 80 and 200 chairs respectively. The swarm is partitioned into four groups using balanced as well as unbalanced partitioning. Then the total time required to complete the task is compared and analysed for both partitioning techniques. “Figure 8”, “figure 9” and “figure 10” plots the number of chairs to be collared along the x-axis and the time in minutes along the y-axis.

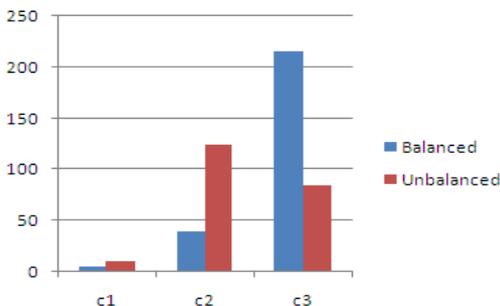


Figure 9. Represents problem 2

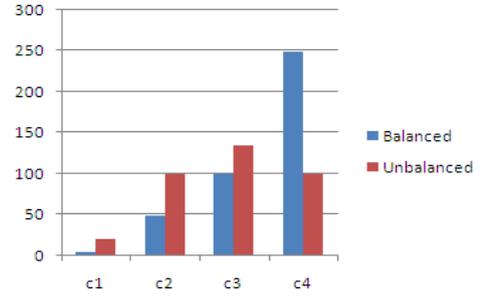


Figure 10. Represents problem 3

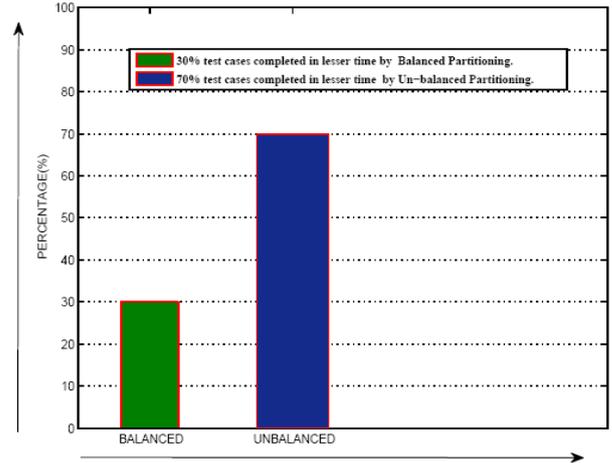


Figure 11. Statistical representation

Result: Out of the 50 test cases analysed, 70 percent of the cases prefer un-balanced partitioning and 30 percent of the cases support balanced partitioning with respect to time. “Figure 11”, shows the statistical representation of cases for both balanced and un-balanced partitioning techniques respectively.

The following conclusions can be made from the experiments carried out:

(i) Un-balanced partitioning is advantageous for uneven task distribution where the task difference between the groups is huge. In such situations, the time taken to complete the total task is more in comparison to un-balanced partitioning.

(ii) Algorithm U\_PART is useful when the task distribution between the groups increases in ascending order with huge difference between each group.

(iii) With respect to time, un-balanced partitioning gains advantage over balanced partitioning, as the value of K increases and when the last group contains huge task load compared to other groups.

(iv) Balanced partitioning is not possible for any prime value of N and also for all N, K pairs where K does not divides N. As there are no such restrictions for un-balanced partitioning, hence U\_PART algorithm partitions N for a wide range of values K.

Thus it can be concluded that for certain applications, where un-even task distribution is required; un-balanced partitioning is more advantageous compared to balanced partitioning.

## 6. An Alternative Approach to Unbalanced Partitioning

### 6.1. Limitation of U\_PART Algorithm

Although, algorithm U\_PART works well for a large value of N, K pairs but it limits the members in each group to its group number except the last group, for all values of N.

This limitation has been overcome in the next section, where an alternative approach has been taken into consideration towards un-balanced partitioning and an un-balanced algorithm U\_PART II has been proposed.

### 6.2. Algorithm: Unbalanced Partitioning II

This section proposes another algorithm to partition N number of robots into K number of unbalanced groups. All assumptions are same as the first algorithm. On being active each robot uses an array  $m[50]$  which stores the number of members present in each group and a variable  $s$  which stores the sum of the members in each group, except the last group.

#### Algorithm U\_PARTII (Executed by Robot R)

**Look:** The Look step remains same as in algorithm U\_PART.

#### Compute:

**Step 1:** This step remains same as in algorithm U\_PART.

**Step 2:** Next, robot R checks whether  $(K=2)$ , if true then there are only two groups. Member in  $Gr_1$  is  $m_1=(N/2)$  and  $Gr_2$  is  $m_2=(N-m_1)$ , where  $m_1$  and  $m_2$  (which stores only integer values and the fractional part if any is truncated), represents the number of members in  $Gr_1$  and  $Gr_2$  respectively. Next, it is checked if  $(m_1=m_2)$ ; if true then set  $m_1=m_1-1$  and set  $(m_2=m_2+1)$ . Otherwise go to step 3.

**Step 3:** If  $(K \neq 2)$  then robot R initializes  $i=0$ ; and for each value of  $i$ , it sets  $m[i]=(N/K-i)$  ( $m[i]$  stores only integer values and the fractional part  $s$  are truncated) and increments the value ( $i=i+1$ ) till  $(i < K-1)$ .  $m[i]$  gives the number of members for  $(i+1)^{th}$  group,  $Gr_{i+1}$ .

**Step 4:** R initializes  $s=0$ ; and stores the sum of all  $m[i]$  values in variable  $s$  for all values of  $i$ ; it updates  $s=s+m[i]$  for every value of  $i$  and checks whether  $s$  exceeds  $N$ ; i.e.;  $(s < N)$  till  $(i < K-1)$ . If true, then  $ml=N-s$  where  $ml$  stores the number of members in the  $K^{th}$  group,  $Gr_K$ . Otherwise go to step 5.

**Step 5:** If  $(s \geq N)$  is true then R decides that only  $(K-1)$  groups are possible and updates  $s=s-m[i]$  and assigns  $ml=N-s$ ; where  $ml$  represents the number of members in the  $(K-1)^{th}$  group,  $Gr_{K-1}$ .

**Step 6:** After robot R calculates the number of members in each group it then decides its own group number. R first initializes  $i=0$ ; If the robot is having rank "p", it checks whether  $p \leq m[i]$ , if true it then sets the  $FLAG=i+1$ . If false it increments the value of  $i$  by 1 and checks if  $p \leq m[i]+m[i+1]$ . If true it sets  $FLAG=i+2$ ; for each false outcome the value of  $i$  is incremented by 1 and the rank  $p$  is checked against the previous cumulative sum added to the next group members. The checking continues till  $i < (K-1)$ , where  $K$  groups are possible (otherwise it continues till  $i < (K-2)$ ). If false R is allocated to  $K^{th}$  group and sets its  $FLAG=K$ . Otherwise it

belongs to  $Gr_{K-1}$ ;  $FLAG=K-1$  for  $(K-1)$  groups.

**Step 7:** This step remains same as in algorithm U\_PART step 5.

**Move:** The move step remains same as in algorithm U\_PART.

### 6.3. Correctness Proofs for U\_Part II

#### 6.3.1. Observations

All the observations mentioned in algorithm U\_PART remains same and are all valid for algorithm U\_PARTII. The algorithm U\_PARTII guarantees un-balanced partitioning. Let us consider the following cases below:

Case 1: Let  $N=10$ , i.e.; robots present in the field and  $K=2$ . Then by the algorithm U\_PARTII, the 10 robots ranked from 1 to 10. As  $K=2$ , by U\_PARTII;  $m_1=(N/2)=10/2=5$ ;  $m_2=N-5=5$ ; now since  $(m_1=m_2)$  we set  $m_1=m_1-1$  and  $m_2=m_2+1$ ; hence  $m_1=5-1=4$ ;  $m_2=5+1=6$ . Robot  $R_1, R_2, R_3, R_4$  is assigned group  $Gr_1$  as rank  $p \leq m_1$  and  $R_5, R_6, R_7, R_8, R_9, R_{10}$  belongs to  $Gr_2$  by algorithm U\_PARTII.

Case 2: Let  $N=25$  and  $K=3$ . Then by algorithm U\_PARTII, the 25 robots are ranked from 1 to 25. As here  $K > 2$ , by U\_PARTII;  $i=0, s=0$ ; as  $m[i]=(N/K-i)$ ;  $m[1]=25/(3-0)=8$  (taking only the truncated value);  $m[2]=25/(3-1)=25/2=12$  and  $s=s+m[0]+m[1]=20$ ; as  $(20 < N)$  henceforth  $ml=N-s=25-20=5$ ; therefore  $Gr_{i+1}=Gr_i$  contains 8 members; similarly  $Gr_2$  contains 12 members and  $Gr_3$  contains 5 members. Robot  $R_1, R_2, R_3, \dots, R_8$  is assigned group  $Gr_1$  as rank  $p \leq m[0]$  and  $R_9, R_{10}, R_{11}, \dots, R_{20}$  belongs to  $Gr_2$  since  $p \leq m[0]+m[1]=12+8=20$  and finally  $R_{21}, R_{22}, R_{23}, \dots, R_{25}$  belongs to  $Gr_3$ .

6.3.2. Result: The partitioning algorithm U\_PARTII will be completed within a finite amount of time

### 6.4. Limitation of Algorithm U\_PARTII

The U\_PARTII algorithm overcomes the shortcomings of U\_PART algorithm by allocating members to groups which are not fixed for all values of N. But it does not guarantee partitions for a wide range of values of K. Let us take up two examples to bring out the limitation:

Case 1: For  $N=10$  and  $K=2$ ;  $Gr_1$  and  $Gr_2$  contains  $m_1=4$  and  $m_2=6$  members each.

Case 2: For  $N=25$  and  $K=5$ ; Members in group 1 = 5; Members in group 2 = 6; Members in group 3 = 8; Members in group 4 = 6. For this value of  $n=25$ , only 4 groups are possible.

Thus U\_PARTII algorithm guarantees un-balanced partitioning but does not guarantee partitioning for all values of K. We may try to overcome this limitation in our future work.

## 7. Conclusions

The development in the field of swarm robotics is reaching a point where various new applications are emerging. Here we have dealt with the partitioning problem where

swarm of autonomous robots are balanced partitioned into equal groups of robots and have proposed our own partitioning techniques for un-balanced partitioning.

In case of un-balanced task allocation the balanced partitioning becomes disadvantageous from the time consuming point of view, thus in such an environment unbalanced partitioning has been introduced, where a swarm of robots are partitioned into  $K$  unbalanced groups.

The algorithm U\_PART partitions a swarm of robots in an unbalanced way over a square field. If the area considered is rectangle or polygonal in shape, then the partitioning can be done in similar manner. To bring out the benefits of the algorithm U\_PART, a simulator has been created, where 50 experiments have been carried out. Based on the results obtained from the 50 test cases, it can be concluded that U\_PART algorithm performs better than the balanced algorithm for certain applications which uses uneven task distribution techniques.

As U\_PART algorithm allocates fixed number of robots for each group except the last group; for all values of  $N, K$ , we have proposed another alternative approach towards un-balanced partitioning to overcome this limitation via algorithm U\_PARTII. Although the algorithm U\_PARTII overcomes the limitation of algorithm U\_PART but does not guarantees partitioning for all values of  $K$ .

Henceforth, it may be concluded that the two alternative approaches for unbalanced partitioning certainly have their own limitations, but at the same time are useful for particular purposes and can be used alternatively as the situation demands.

## 8. Future Work

This piece of research work proposes two un-balanced partitioning approach on a swarm of autonomous mobile robots. Here two algorithms U\_PART and U\_PARTII have been proposed which overcomes the shortcomings of the balanced algorithm, where  $N$  robots have been partitioned into  $K$  unbalanced groups. Here a square field has been considered over which the robots are placed for partitioning; the algorithm can also be performed over a rectangle or polygonal field of known or unknown dimension, in similar manner.

Both the algorithms have certain limitations which may be taken up as future scopes of study: (i) Overcoming the limitation on fixed number of members in a group in the algorithm U\_PART. (ii) Overcoming the limitation on partitioning for wide range of  $K$  values in the algorithm U\_PARTII. (iii) Moreover, an obstacle-less field has been considered for both the techniques. In practical cases there may be situations where the robots are placed in a field where obstacles are present. It may be aimed at devising an algorithm in future for unbalanced partitioning which will work in such environment also. (iv) The robots considered here has unlimited visibility capabilities. In future research work, the algorithm may be extended to support limited

visibility of the robots as well.

## ACKNOWLEDGEMENTS

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## APPENDIX A

**Table A.1.** Result of Simulation: 35 test cases supporting unbalanced partitioning

Test Cases	$C_i$ =Task in each group	$T_{ib}$	$T_{iub}$	TOB	TOUB
Case 1: N=10, K=2	10 110	10 110	50 65	110	65
Case 2: N=10, K=2	23 322	25 325	115 180	325	180
Case 3: N=12, K=3	2 20 100	5 25 125	10 50 60	125	60
Case 4: N=12, K=4	2 20 50 100	5 35 85 170	10 50 85 85	170	85
Case 5: N=15, K=3	3 30 90	5 30 90	15 75 40	90	75
Case 6: N=16, K=4	4 40 80 200	5 50 100 250	20 100 135 100	250	135
Case 7: N=20, K=5	14 35 55 75 105	20 45 70 95 135	70 90 95 95 55	135	95
Case 8: N=21, K=3	2 50 300	5 40 215	10 125 85	215	125
Case 9: N=25, K=5	1 5 15 25 55	5 5 15 25 55	5 15 25 35 20	55	35
Case 10: N=30, K=5	20 40 60 100 200	20 35 50 85 170	100 100 125 50	170	125
Case 11: N=20, K=2	10 160	5 80	50 45	80	50
Case 12: N=12, K=3	2 28 300	5 35 375	10 70 170	375	170
Case 13: N=18, K=3	3 10 200	5 10 170	15 25 70	170	70
Case 14: N=27, K=3	2 8 100	5 5 60	10 20 25	60	25

Case 15: N=27, K=3	5 10 95	5 10 55	25 25 20	55	25
Case 16: N=27, K=3	5 25 300	5 15 170	25 65 65	170	65
Case 17: N=50, K=5	2 5 10 11 90	5 5 5 10 45	10 15 20 15 15	45	20
Case 18: N=111, K=3	1 2 50	5 5 10	5 5 5	10	5
Case 19: N=9, K=3	2 3 9	5 5 15	10 10 10	15	10
Case 20: N=6, K=3	3 5 7	10 15 20	15 15 15	20	15
Case 21: N=10, K=2	2 170	5 170	10 95	170	95
Case 22: N=15, K=5	2 5 10 30 50	5 10 20 50 85	10 15 20 40 50	85	50
Case 23: N=90, K=10	9 10 22 33 44 55 66 77 88 99	5 10 15 20 25 35 40 45 50 55	45 25 40 45 50 50 50 50 15	55	50
Case 24: N=80, K=10	8 10 20 30 40 50 60 70 80 90	5 10 15 20 25 35 40 45 50 60	40 25 40 45 45 45 15	60	45
Case 25: N=70, K=10	7 20 30 40 50 60 70 80 90 100	5 15 25 30 40 45 50 60 65 75	35 50 50 50 50 50 50 50 50 20	75	50
Case 26: N=21, K=3	2 23 333	5 20 240	10 60 95	240	95
Case 27: N=15, K=5	2 3 5 55 66	5 5 10 95 110	10 10 10 70 70	110	70
Case 28: N=60, K=10	2 3	5 5	10 10	15	10

	4 6 8 10 11 14 15 17	5 5 10 10 10 15 15 15	10 10 10 10 10 10 10		
Case 29: N=70, K=10	1 2 3 4 5 6 7 8 9 10	5 5 5 5 5 5 5 10 10 5	5 5 5 5 5 5 5 5 5 5	10	5
Case 30: N=80, K=10	1 2 3 4 5 6 7 8 9 10	5 5 5 5 5 5 5 10 10 5	5 5 5 5 5 5 5 5 5 5	10	5
Case 31: N=90, K=10	1 2 3 4 5 6 7 8 9 10	5 5 5 5 5 5 5 5 5 10	5 5 5 5 5 5 5 5 5 5	10	5
Case 32: N=40, K=4	1 3 5 50	5 5 5 25	5 10 10 10	25	10
Case 33: N=40, K=5	1 2 3 4 40	5 5 5 5 25	5 5 5 5 10	25	10
Case 34: N=100, K=5	1 3 5 7 70	5 5 5 5 20	5 10 10 10 5	20	10
Case 35: N=16, K=4	1 2 20 40	5 5 25 50	5 5 35 20	50	35

**Table A.2.** Result of Simulation: 15 test cases supporting balanced partitioning

Test Cases	$C_i$ =Task in each group	$T_{ib}$	$T_{iub}$	TOB	TOUB
Case 1: N=12, K=3	2 5	5 10	10 15	10	15

	7	10	5		
Case 2: N=21, K=3	5	5	25	25	40
	15	5	40		
	35	25	10		
Case 3: N=12, K=3	7	5	35	75	125
	50	40	125		
	100	75	30		
Case 4: N=28, K=4	4	5	20	10	20
	7	5	20		
	8	10	15		
	10	10	5		
Case 5: N=30, K=3	20	10	100	20	100
	30	15	75		
	40	20	10		
Case 6: N=40, K=4	10	5	50	35	85
	30	15	75		
	50	25	85		
	70	35	15		
Case 7: N=40, K=5	10	10	50	90	125
	40	25	100		
	70	45	120		
	100	65	125		
Case 8: N=100, K=5	100	25	500	125	500
	200	50	500		
	300	75	500		
	400	100	500		
	500	125	30		
Case 9: N=200, K=10	5	5	25	255	1045
	25	10	65		
	625	160	1045		
	725	185	910		
	825	210	825		
	925	235	775		
	1000	250	715		
	1005	255	630		
	1010	255	565		
	1020	255	35		
Case 10: N=300, K=3	2	55	10	5	10
	4	5	10		
	70	5	5		
Case 11: N=300, K=3	5	5	25	150	230
	50	10	125		
	55	10	95		
	67	15	85		
	87	15	90		
	97	20	85		
	107	2	80		
	207	35	130		
	407	70	230		
	900	150	20		
Case 12: N=300, K=10	2	5	10	150	335
	20	5	50		
	50	10	85		
	80	15	100		
	100	20	100		
	150	25	125		
	200	35	145		
	500	85	315		
	600	100	335		
	900	150	20		
Case 13: N=27, K=3	5	5	25	60	65
	25	15	65		
	105	60	25		

Case 14: N=120, K=3	2	5	10	20	100
	40	5	100		
	160	20	10		
Case 15: N=48, K=4	5	5	25	50	125
	50	25	125		
	60	25	100		
	120	50	15		

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